AI Art at Christie’s Sells for $432,500
Photometric Stereo

Computer Vision
Fall 2018
Columbia University
Last Time: Two-View Stereo

Key Idea: use feature motion to understand shape
Today: Photometric Stereo

Key Idea: use pixel brightness to understand shape
Today: Photometric Stereo

Key Idea: use pixel brightness to understand shape
Foreshortening

As the surface tilts away from the light source the same light energy is spread over a larger area, making the surface darker.
Photometric Stereo

What results can you get?

Input (1 of 12)  Normals (RGB colormap)  Normals (vectors)  Shaded 3D rendering  Textured 3D rendering
Modeling Image Formation

Now we need to reason about:

• How light interacts with the scene
• How a pixel value is related to light energy in the world

Track a “ray” of light all the way from light source to the sensor
Light rays interacting with a surface

• Light of radiance $L_i$ comes from light source at an incoming direction $\theta_i$
• It sends out a ray of radiance $L_r$ in the outgoing direction $\theta_r$
• How does $L_r$ relate to $L_i$?

- N is surface normal
- L is direction of light, making $\theta_i$ with normal
- V is viewing direction, making $\theta_r$ with normal
Light rays interacting with a surface

- $N$ is surface normal
- $L$ is direction of light, making $\theta_i$ with normal
- $V$ is viewing direction, making $\theta_r$ with normal

Output radiance along $V$

$$L_r = \rho(\theta_i, \theta_r) L_i \cos \theta_i$$

Incoming irradiance along $L$

Bi-directional reflectance function (BRDF)
Materials - Three Forms

- Ideal diffuse (Lambertian)
- Ideal specular
- Directional diffuse
Reflectance—Three Forms

- Ideal diffuse (Lambertian)
- Directional diffuse
- Ideal specular
Ideal Diffuse Reflection

• Characteristic of multiple scattering materials
• An idealization but reasonable for matte surfaces
• Basis of most radiosity methods
Ideal Specular Reflection

- Calculated from Fresnel’s equations
- Exact for polished surfaces
- Basis of early ray-tracing methods
Light rays interacting with a surface

\[ L_r = \rho(\theta_i, \theta_r)L_i \cos \theta_i \]

- Special case 1: Perfect mirror
  - \[ \rho(\theta_i, \theta_r) = 0 \text{ unless } \theta_i = \theta_r \]
- Special case 2: Matte surface
  - \[ \rho(\theta_i, \theta_r) = \rho_0 \text{ (constant)} \]
For now, ignore specular reflection
And Refraction…
And Interreflections…

Slides from Photometric Methods for 3D Modeling, Matsushita, Wilburn, Ben-Ezra
And Subsurface Scattering...
Lambertian surfaces

• For a lambertian surface:

\[ L_r = \rho L_i \cos \theta_i \]

\[ \Rightarrow L_r = \rho L_i L \cdot N \]

• \( \rho \) is called \textit{albedo}
  • Think of this as paint
  • High albedo: white colored surface
  • Low albedo: black surface
  • Varies from point to point
Lambertian surfaces

- Assume the light is directional: all rays from light source are parallel
  - Equivalent to a light source infinitely far away

- All pixels get light from the same direction \( \mathbf{L} \) and of the same intensity \( L_i \)
Lambertian surfaces

\[ I(x, y) = \rho(x, y) L_i L \cdot N(x, y) \]

Reflectance image

Shading image

Intrinsic Image Decomposition
Reconstructing Lambertian surfaces

\[ I(x, y) = \rho(x, y) L_i \cdot \mathbf{L} \cdot \mathbf{N}(x, y) \]

- Equation is a constraint on albedo and normals
- Can we solve for albedo and normals?
Solution 1: Recovery from a single image

• Step 1: Intrinsic image decomposition
  • Reflectance image $L_i$
  • Shading image
  • Decomposition relies on priors on reflectance image
  • What kind of priors?
  • Reflectance image captures the "paint" on an object surface
  • Surfaces tend to be of uniform color with sharp edges when color changes

Images from Barron et al, TPAMI 13
Solution 1: Recovery from a single image

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- Step 1: Intrinsic image decomposition
  - Reflectance image
  - Shading image

Decomposition relies on priors on the reflectance image.

What kind of priors?

- Reflectance image captures the "paint" on an object surface.
- Surfaces tend to be of uniform color with sharp edges when color changes.

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Solution 1: Recovery from a single image

Step 1: Intrinsic image decomposition

- Reflectance image
- Shading image

Decomposition relies on priors on reflectance image

What kind of priors?

- Reflectance image captures the "paint" on an object surface
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Images from Barron et al, TPAMI 13
Solution 1: Recovery from a single image

• Step 1: Intrinsic image decomposition
  • Reflectance image $L_i$
  • Shading image $L \cdot N(x, y)$

Decomposition relies on priors on reflectance image

• What kind of priors?
  • Reflectance image captures the "paint" on an object surface
  • Surfaces tend to be of uniform color with sharp edges when color changes

Images from Barron et al, TPAMI 13
Solution 1: Recovery from a single image

- Step 1: Intrinsic image decomposition
  - Reflectance image $\rho(x, y)$
  - Shading image $L_i L \cdot N(x, y)$
  - Decomposition relies on priors or reflectance image

- What kind of priors?
  - Reflectance image captures the “paint” on an object surface
  - Surfaces tend to be of uniform color with sharp edges when color changes

Images from Barron et al, TPAMI 13
Solution 1: Recovery from a single image

• Step 2: Decompose shading image into illumination and normals

\[ L_i \mathbf{L} \cdot \mathbf{N}(x, y) \]
• Called Shape-From-Shading
• Relies on priors on shape: shapes are smooth
Photometric Stereo

Key Idea: use pixel brightness to understand shape
Solution 2: Recovery from multiple images

\[ I(x, y) = \rho(x, y) L_i L \cdot N(x, y) \]

- Represents an equation in the albedo and normals
- Multiple images give constraints on albedo and normals
- Called *Photometric Stereo*

Photometric stereo - the math

\[ I(x, y) = \rho(x, y) L_i L \cdot N(x, y) \]

• Consider single pixel
• Assume \( L_i = 1 \)

\[ I = \rho L \cdot N \]

\[ I = \rho N^T L \]

• Write \( G = \rho N \)
• \( G \) is a 3-vector
  • Norm of \( G = \rho \)
  • Direction of \( G = N \)
Photometric stereo - the math

- Consider single pixel
- Assume $L_i = 1$
  \[ I = \rho N^T L \]
- Write $G = \rho N$
- $G$ is a 3-vector
  - Norm of $G = \rho$
  - Direction of $G = N$
  \[ I = G^T L = L^T G \]
Photometric stereo - the math

\[ I = L^T G \]

- Multiple images with different light sources but same viewing direction?

\[ I_1 = L_1^T G \]

\[ I_2 = L_2^T G \]

\[ \vdots \]

\[ I_k = L_k^T G \]
Photometric stereo - the math

- Assume lighting directions are known
- Each is a linear equation in $G$
- Stack everything up into a massive linear system of equations!

\[
\begin{align*}
I_1 &= \mathbf{L}_1^T \mathbf{G} \\
I_2 &= \mathbf{L}_2^T \mathbf{G} \\
\vdots \\
I_k &= \mathbf{L}_k^T \mathbf{G}
\end{align*}
\]
Photometric stereo - the math

\[ I_1 = L_1^T G \]
\[ I_2 = L_2^T G \]
\[ \vdots \]
\[ I_k = L_k^T G \]

\[ \mathbf{I} = \mathbf{L}^T \mathbf{G} \]

- \( k \times 3 \) matrix of lighting directions
- \( k \times 1 \) vector of intensities
- 3x1 vector of unknowns
Photometric stereo - the math

\[ I = L^T G \]

\[ G = L^{-T} I \]

• What is the minimum value of \( k \) to allow recovery of \( G \)?
• How do we recover \( G \) if the problem is overconstrained?
Photometric stereo - the math

• How do we recover $G$ if the problem is overconstrained?
  • More than 3 lights: more than 3 images

• Least squares

$$\min_G \| I - L^T G \|^2$$

• Solved using normal equations

$$G = (LL^T)^{-1} LI$$
Normal equations

\[ \| I - L^T G \|^2 = I^TI + G^TLL^TG - 2G^TLI \]

- Take derivative with respect to $G$ and set to 0

\[ 2LL^TG - 2LI = 0 \]

\[ \Rightarrow G = (LL^T)^{-1}LI \]
Estimating normals and albedo from $G$

- Recall that $G = \rho N$

\[
\|G\| = \rho
\]

\[
\frac{G}{\|G\|} = N
\]
Multiple pixels

• We’ve looked at a single pixel till now
• How do we handle multiple pixels?
• Essentially independent equations!
Multiple pixels: matrix form

• Note that all pixels share the same set of lights

\[
\mathbf{I}^{(1)} = \mathbf{L}^T \mathbf{G}^{(1)} \\
\mathbf{I}^{(2)} = \mathbf{L}^T \mathbf{G}^{(2)} \\
\vdots \\
\mathbf{I}^{(n)} = \mathbf{L}^T \mathbf{G}^{(n)}
\]
Multiple pixels: matrix form

- Can stack these into columns of a matrix

\[
\begin{align*}
I^{(1)} &= LT G^{(1)} \\
I^{(2)} &= LT G^{(2)} \\
&
\vdots \\
I^{(n)} &= LT G^{(n)}
\end{align*}
\]

\[
\begin{bmatrix}
I^{(1)} & I^{(2)} & \ldots & I^{(n)}
\end{bmatrix} = LT \begin{bmatrix}
G^{(1)} & G^{(2)} & \ldots & G^{(n)}
\end{bmatrix}
\]

\[
I = LTG
\]
Multiple pixels: matrix form

$$I = L^T G$$
Determining Light Directions

• Trick: Place a mirror ball in the scene.

• The location of the highlight is determined by the light source direction.
Real-World HDR Lighting Environments

Funston Beach  Eucalyptus Grove
Uffizi Gallery  Grace Cathedral

Lighting Environments from the Light Probe Image Gallery:
http://www.debevec.org/Probes/

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Extreme HDR Image Series

1 sec f/4
1/4 sec f/4
1/30 sec f/4
1/30 sec f/16
1/250 sec f/16
1/1000 sec f/16
1/8000 sec f/16
Extreme HDR Image Series
sun closeup

1 sec f/4

1/4 sec f/4

1/30 sec f/4

1/30 sec f/16

1/250 sec f/16

1/1000 sec f/16

1/8000 sec f/16 only image that does not saturate!
HDRI Sky Probe
Determining Light Directions

- For a perfect mirror, the light is reflected across N:

\[ I_e = \begin{cases} 
I_i & \text{if } \mathbf{V} = \mathbf{R} \\
0 & \text{otherwise} 
\end{cases} \]
Determining Light Directions

So the light source direction is given by:

\[ L = 2(N \cdot R)N - R \]
Photometric Stereo

What results can you get?

Input (1 of 12)  Normals (RGB colormap)  Normals (vectors)  Shaded 3D rendering  Textured 3D rendering
Results

from Athos Georghiades
Unknown Lighting

\[ I = kN \cdot \ell L \]

Surface normals \hspace{1cm} Light directions

Diffuse albedo \hspace{1cm} Light intensity
Unknown Lighting

Surface normals, scaled by albedo

Light directions, scaled by intensity

\[ I = N \cdot L \]
Unknown Lighting

$p = \# \text{ pixels}$

$n = \# \text{ images}$

$I = L^T G$
Unknown Lighting

Measurements (one image per row) \[ I \]

Light directions (scaled by intensity) \[ L^T \]

Surface normals (scaled by albedo) \[ G \]

Both L and G are now unknown! This is a matrix factorization problem.
Unknown Lighting

There’s hope: We know that I is rank 3
Unknown Lighting

Use the SVD to decompose I:

\[ I = UV \]

SVD gives the best rank-3 approximation of a matrix.
Unknown Lighting

Use the SVD to decompose I:

\[ I = U \Sigma V \]

SVD gives the best rank-3 approximation of a matrix. What do we do with \( \Sigma \)?
Unknown Lighting

Use the SVD to decompose $I$:

$$ I = U \sqrt{\Sigma} \sqrt{\Sigma} V $$

Can we just do that?
Unknown Lighting

Use the SVD to decompose I:

\[
I = U\sqrt{\Sigma} A A^{-1} \sqrt{\Sigma} V
\]

Can we just do that? …almost.

The decomposition is unique up to an invertible 3x3 A.
Unknown Lighting

Use the SVD to decompose I:

\[ I = U \sqrt{\Sigma} \begin{bmatrix} A & A^{-1} \end{bmatrix} \sqrt{\Sigma} V \]

Can we just do that? ...almost. \[ L = U \sqrt{\Sigma} A, G = A^{-1} \sqrt{\Sigma} V \]

The decomposition is unique up to an invertible 3x3 \( A \).
Unknown Lighting

Use the SVD to decompose $I$:

$$ I = U \sqrt{\Sigma} A A^{-1} \sqrt{\Sigma} V $$

$L = U \sqrt{\Sigma} A, G = A^{-1} \sqrt{\Sigma} V$

You can find $A$ if you know

• 6 points with the same reflectance, or
• 6 lights with the same intensity.
Unknown Lighting: Ambiguities

- Multiple combinations of lighting and geometry can produce the same sets of images.
- Add assumptions or prior knowledge about geometry or lighting, etc. to limit the ambiguity.

[Belhumeur et al.’97]
Lights, camera, action

Sensor

Lights

Camera
Figure 2. (a) This decorative pin consists of a glass bas-relief portrait mounted in a shiny gold setting. (b) The RGB image provided by the retrographic sensor. The pin is pressed into the elastomer skin, and colored lights illuminate it from three directions.
Figure 7: Comparison with the high-resolution result from the original retrographic sensor. (a) Rendering of the high-resolution $20 bill example from the original retrographic sensor with a close-up view. (b) Rendering of the captured geometry using our method.
Figure 9: Example geometry measured with the bench and portable configurations. Outer image: rendering under direct lighting. Inset: macro photograph of original sample. Scale shown in upper left. Color images are shown for context and are to similar, but not exact scale.