

# Object Tracking

Computer Vision  
Fall 2018  
Columbia University

# Homework 5

- Released last night
- Due November 26th
- Start it today — no extensions!

# Optical Flow

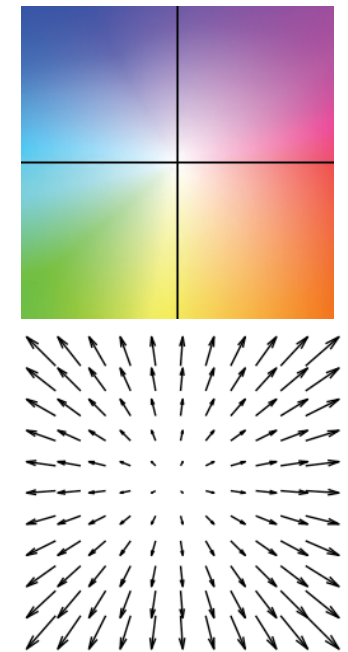
- Optical flow field: assign a flow vector to each pixel
- Visualize: flow magnitude as saturation, orientation as hue



Input two frames



Ground-truth flow field



Visualization code  
[Baker et al. 2007]

# Optical Flow Constraint

$$I_x u + I_y v + I_t = 0$$

- Brightness/color is constant
- Small motions
- Also assume neighboring pixels have same motion



# Solving the aperture problem

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- How to get more equations for a pixel?
- **Spatial coherence constraint:** pretend the pixel's neighbors have the same (u,v)
- If we use a 5x5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p}_i) + \nabla I(\mathbf{p}_i) \cdot [u \ v]$$

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}$$

$$\begin{matrix} A & d = b \\ 25 \times 2 & 2 \times 1 & 25 \times 1 \end{matrix}$$

# Solving the aperture problem

---

Problem: we have more equations than unknowns

$$\begin{array}{ccc} A & d = & b \\ 25 \times 2 & 2 \times 1 & 25 \times 1 \end{array} \longrightarrow \text{minimize } \|Ad - b\|^2$$

# Solving the aperture problem

---

Problem: we have more equations than unknowns

$$\begin{matrix} A & d = b \\ 25 \times 2 & 2 \times 1 & 25 \times 1 \end{matrix} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Solution: solve least squares problem

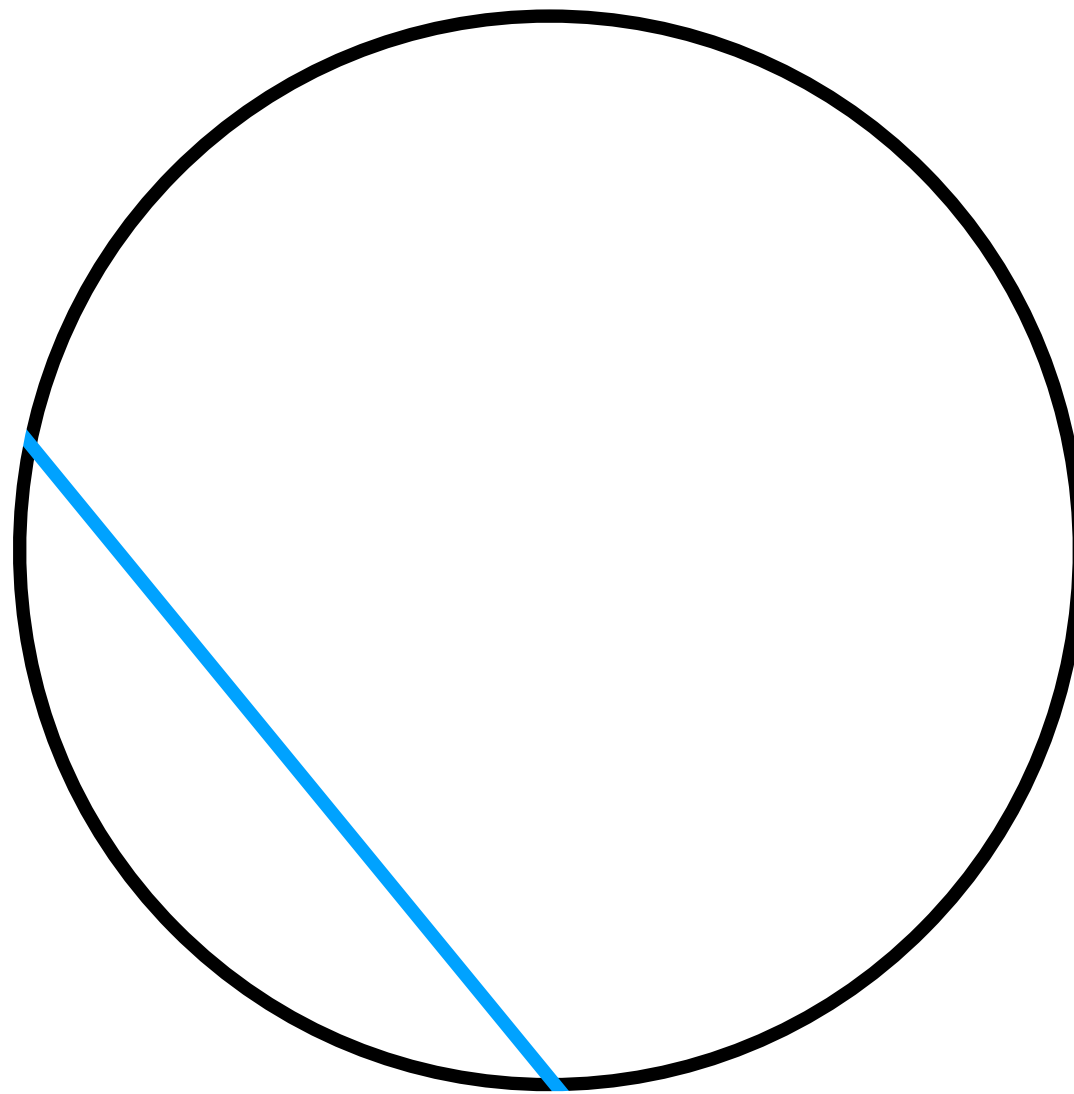
- minimum least squares solution given by solution (in  $d$ ) of:

$$\begin{matrix} (A^T A) & d = A^T b \\ 2 \times 2 & 2 \times 1 & 2 \times 1 \end{matrix}$$

$$\begin{matrix} \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} & \begin{bmatrix} u \\ v \end{bmatrix} = - & \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} \\ A^T A & & A^T b \end{matrix}$$

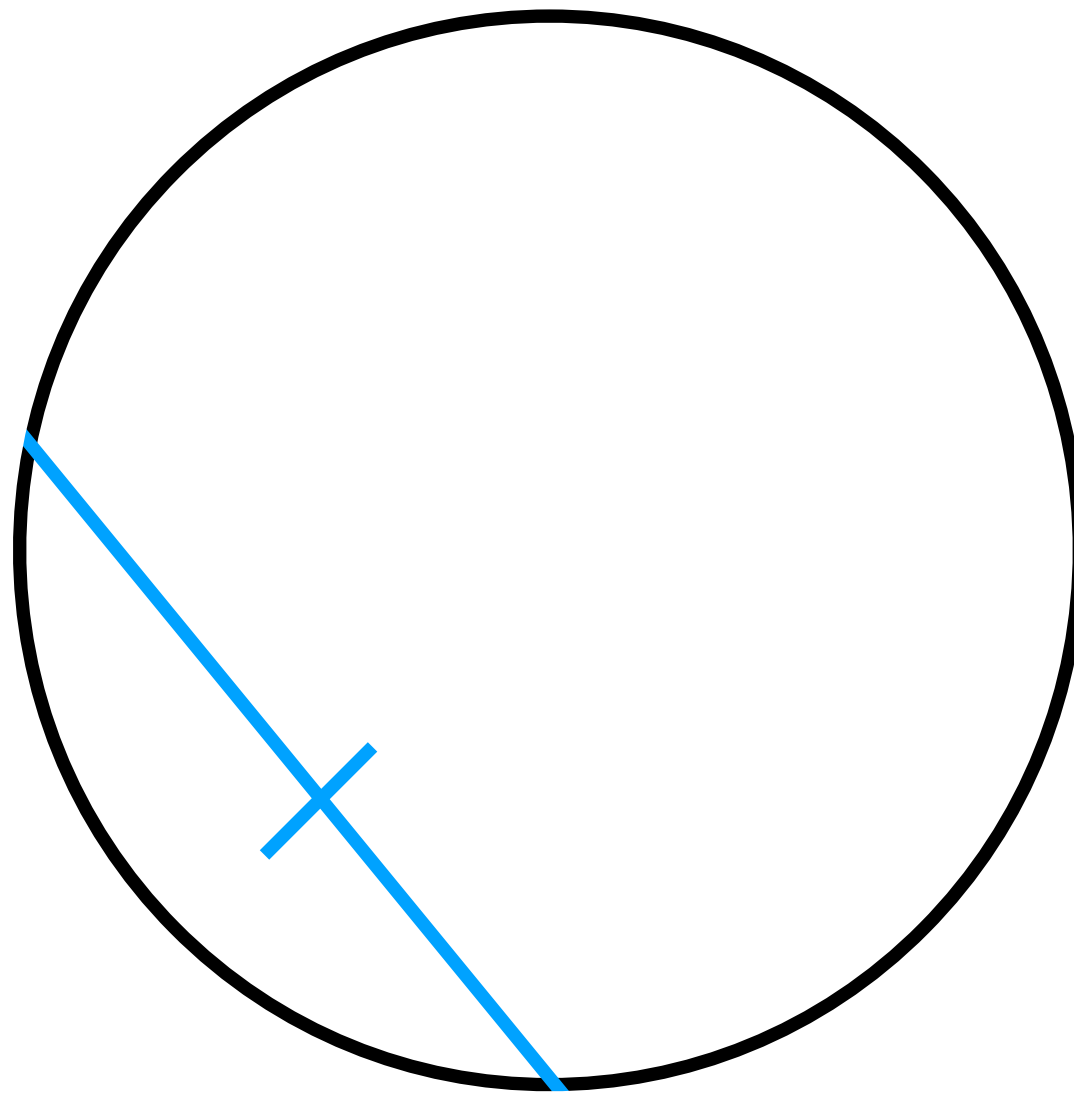
- The summations are over all pixels in the  $K \times K$  window
- This technique was first proposed by Lucas & Kanade (1981)

# Aperture Problem



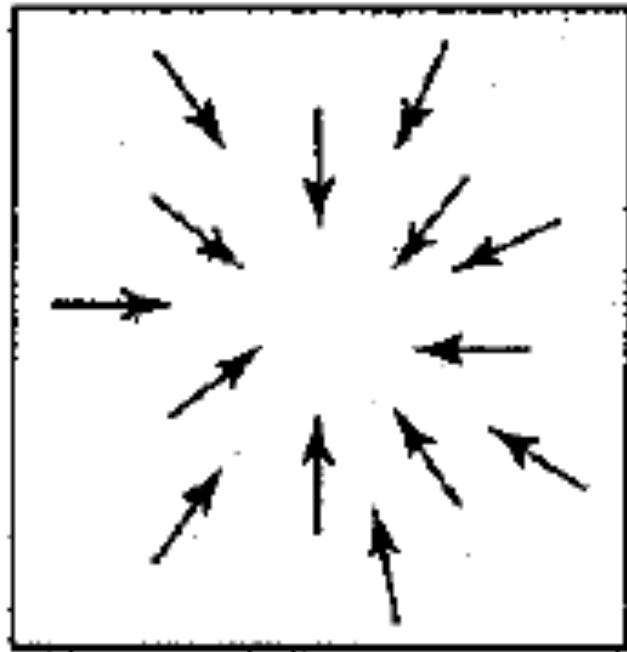
Which way did  
the line move?

# Aperture Problem

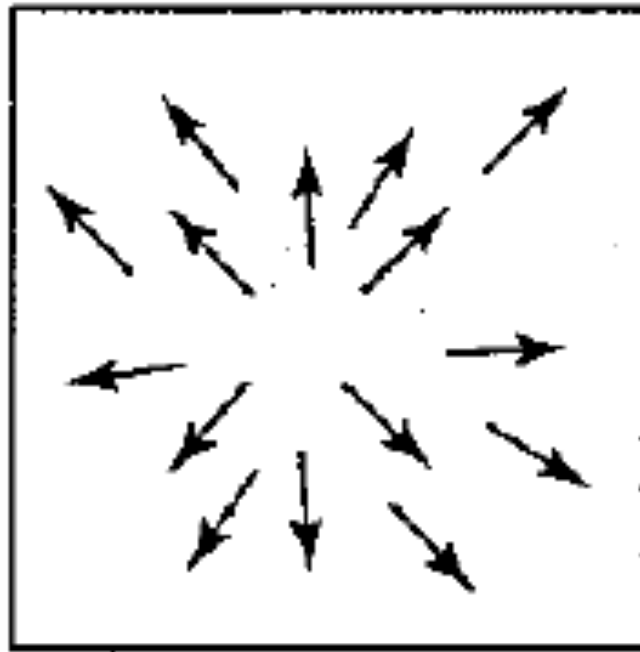


Which way did  
the line move?

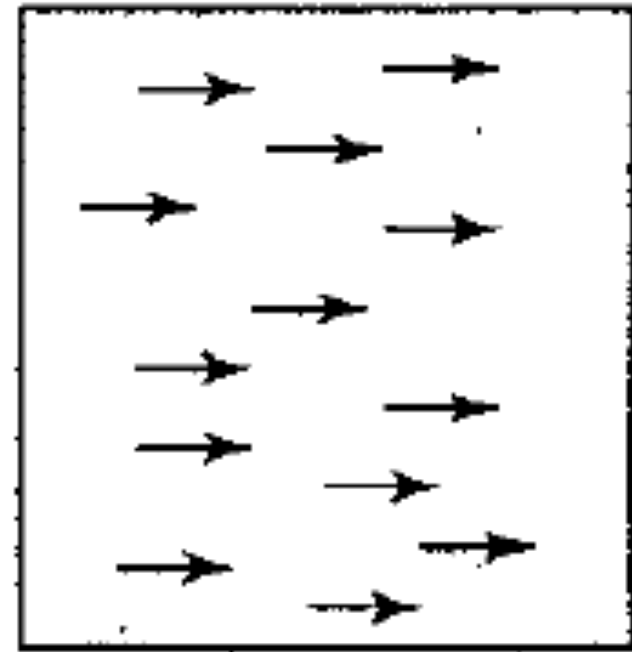
# Motion Fields



**Zoom out**



**Zoom in**

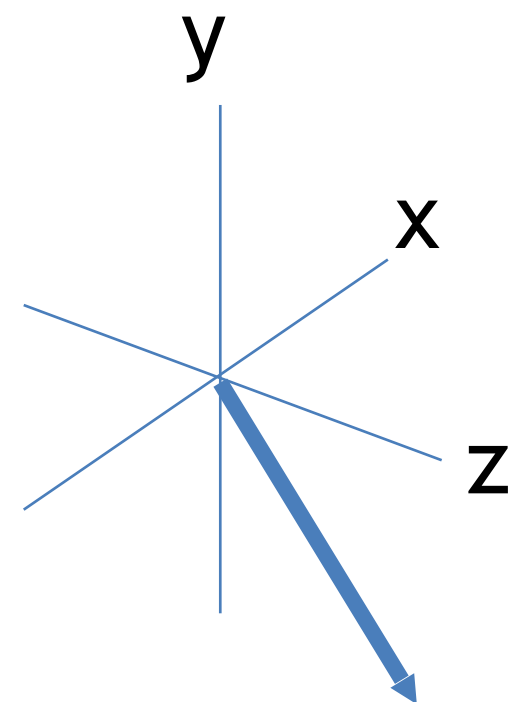
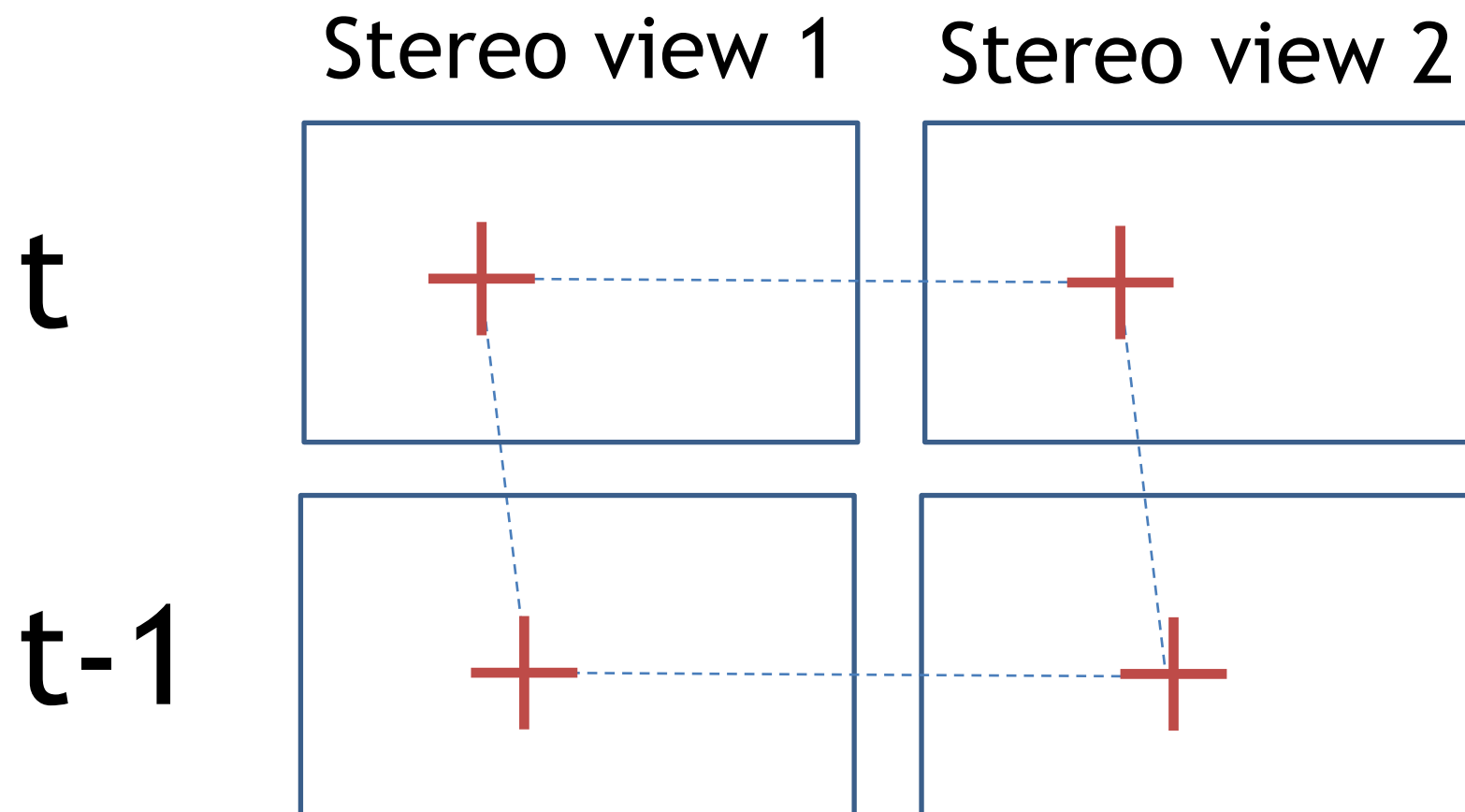


**Pan right to left**

# Can we do more? *Scene flow*

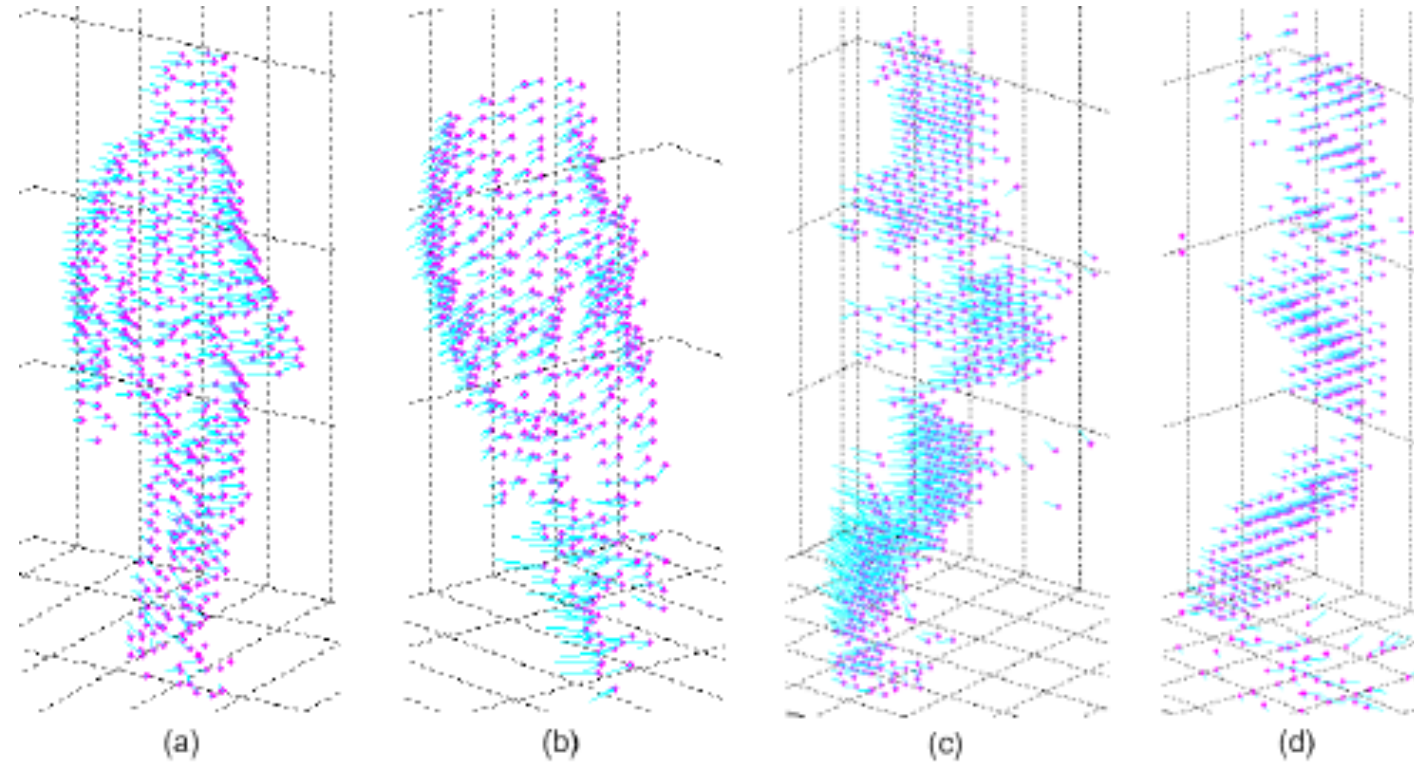
Combine spatial stereo & temporal constraints

Recover 3D vectors of world motion



3D world motion  
vector per pixel

# Scene flow example for human motion



Estimating 3D Scene Flow from Multiple 2D Optical Flows, Ruttle et al., 2009



# Scene Flow

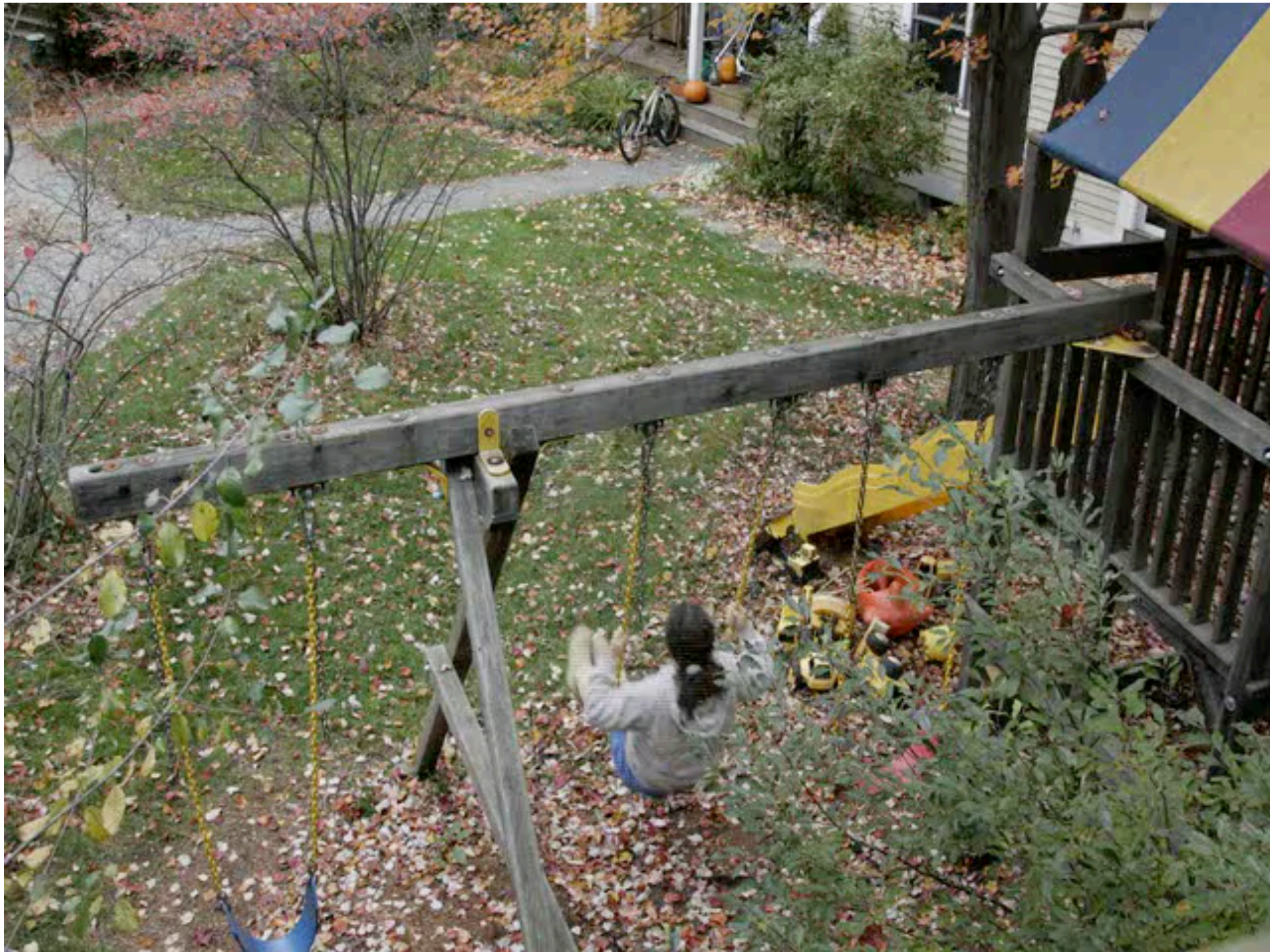
[https://www.youtube.com/watch?v=RL\\_TK\\_Be6\\_4](https://www.youtube.com/watch?v=RL_TK_Be6_4)



<https://vision.in.tum.de/research/sceneflow>



# Motion Analysis



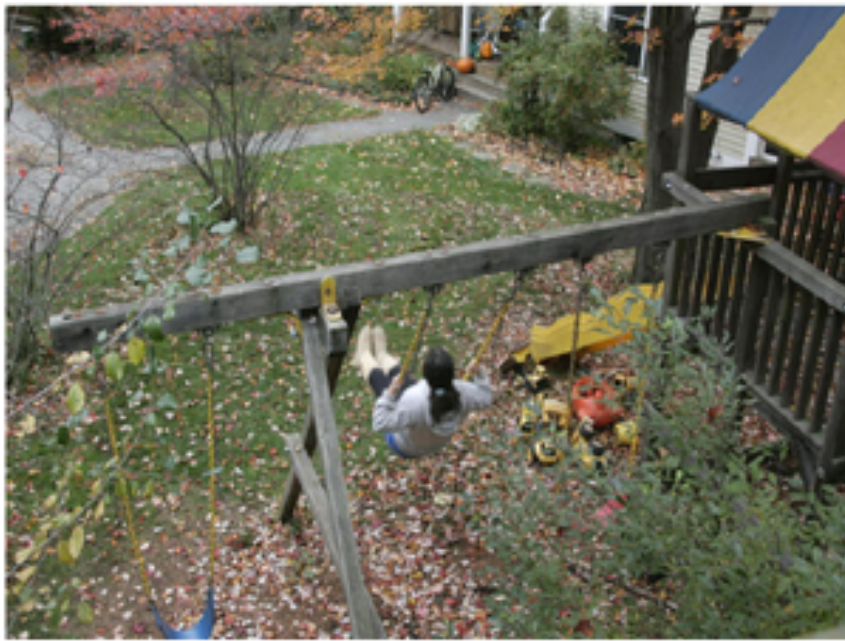


# Motion Magnification





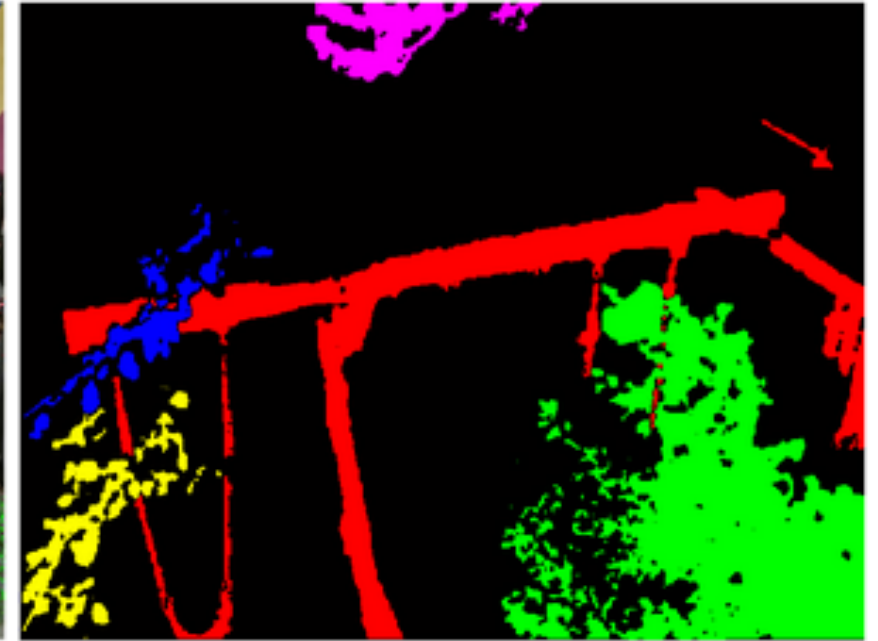
# Motion Magnification



(a) Registered input frame



(b) Clustered trajectories of tracked features



(c) Layers of related motion and appearance



(d) Motion magnified, showing holes



(e) After texture in-painting to fill holes



(f) After user's modification to segmentation map in (c)



# Motion Magnification



# Motion Magnification



Massachusetts Institute of Technology



Revealing Invisible Changes In The World

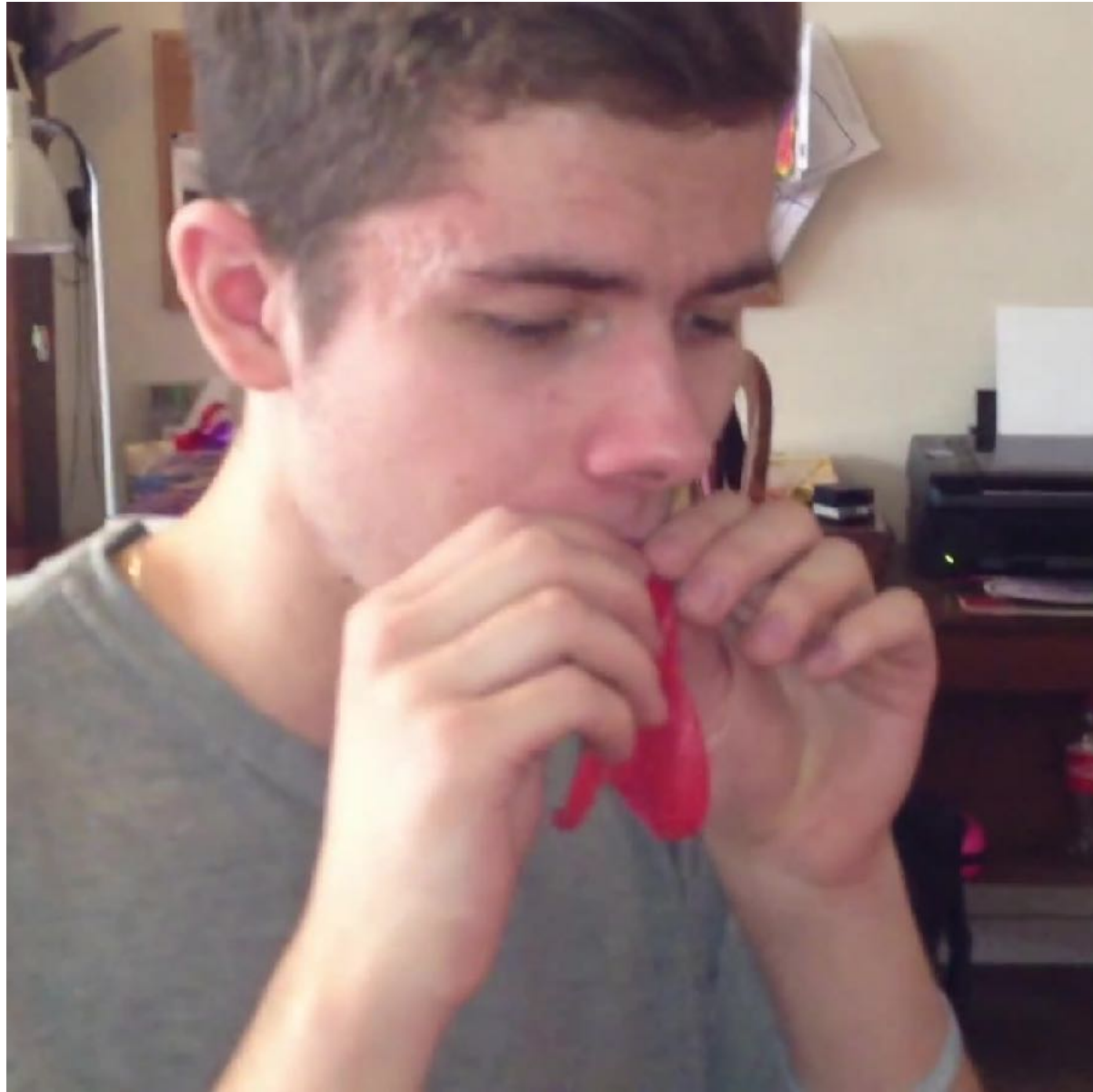
Created for the NSF International Science & Engineering  
Visualization Challenge 2012

# Learning optic flow



Fischer et al. 2015. <https://arxiv.org/abs/1504.06852>

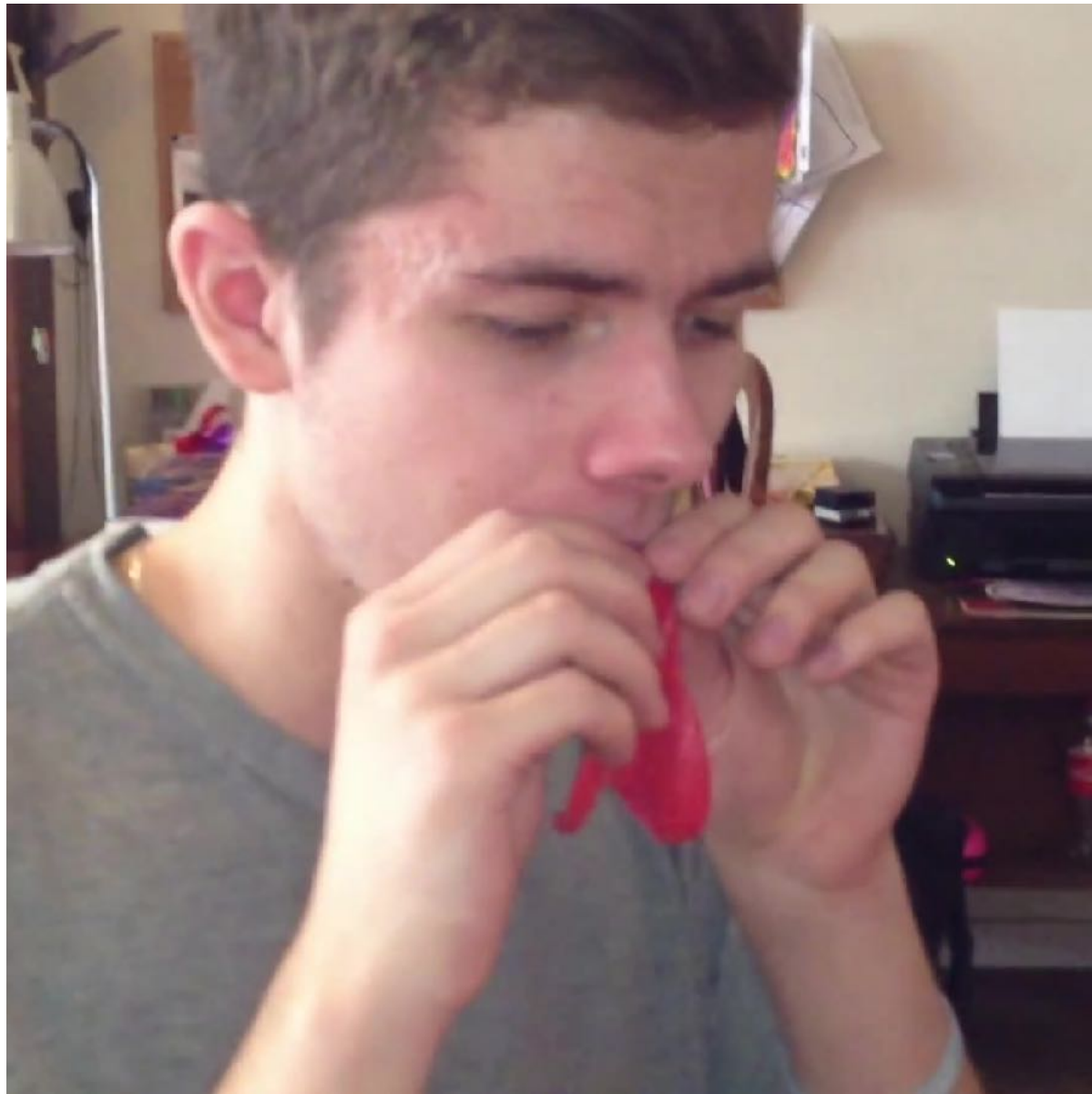




Time 



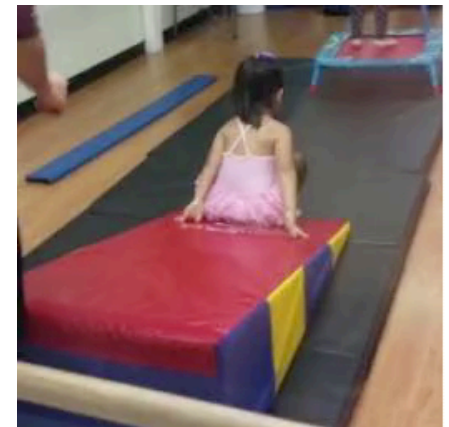
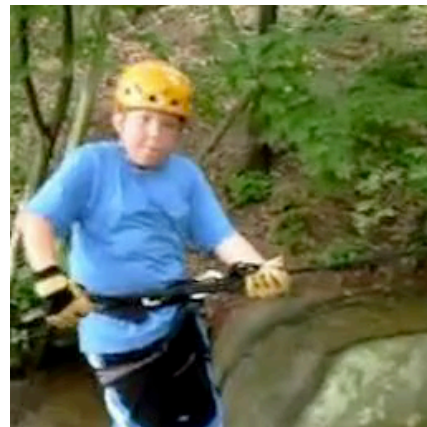
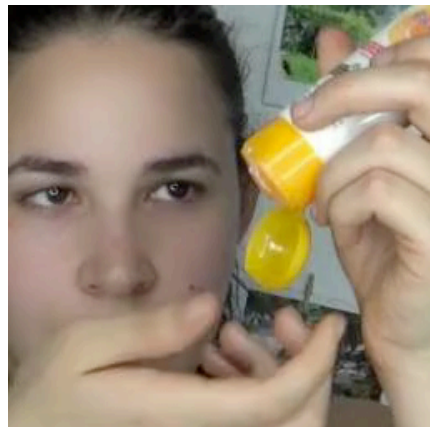
# What color is that pixel?



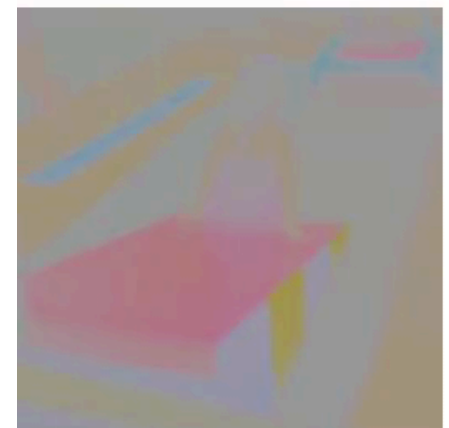
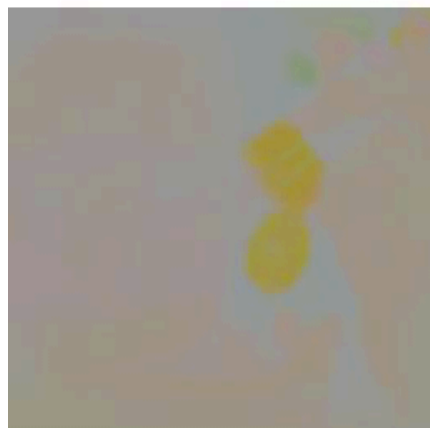
Time 

# Temporal Coherence of Color

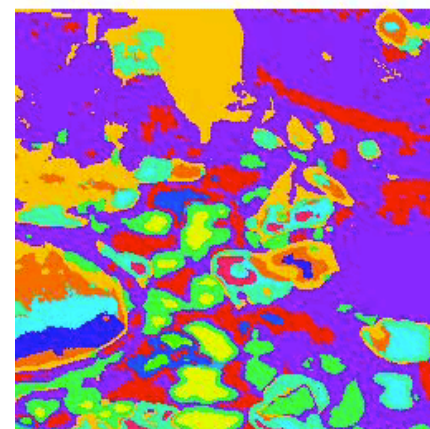
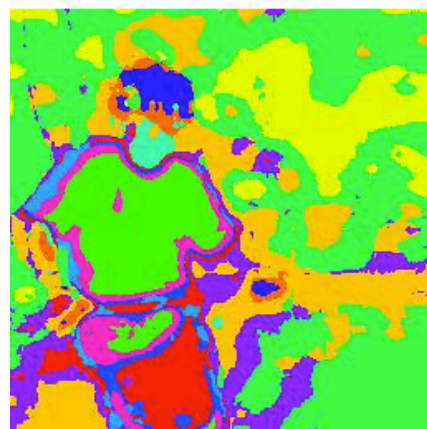
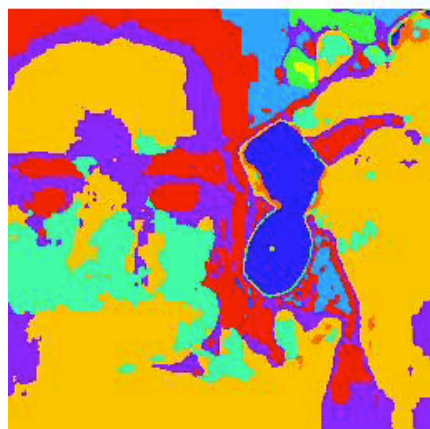
RGB



Color  
Channels



Quantized  
Color



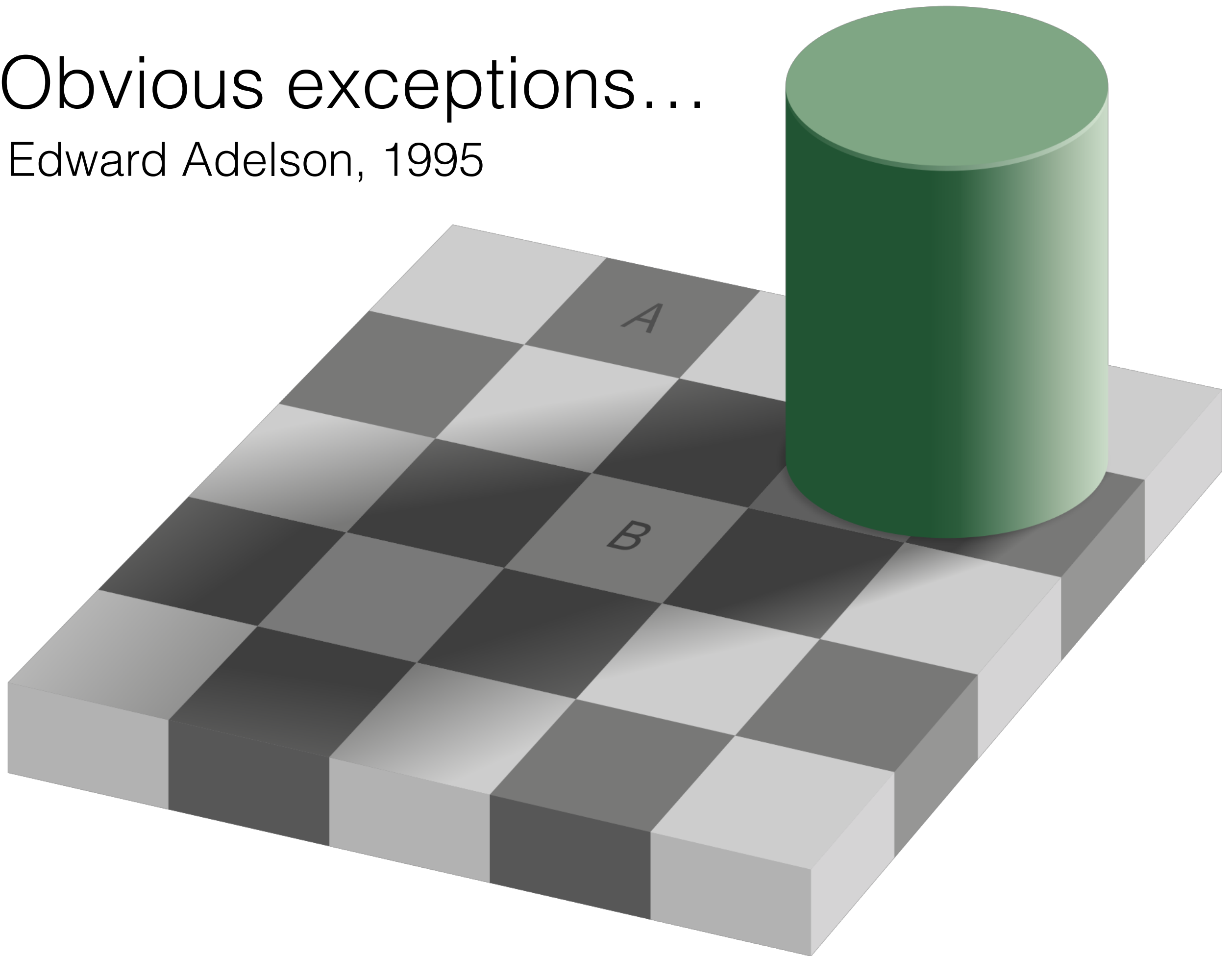


Obvious exceptions...



# Obvious exceptions...

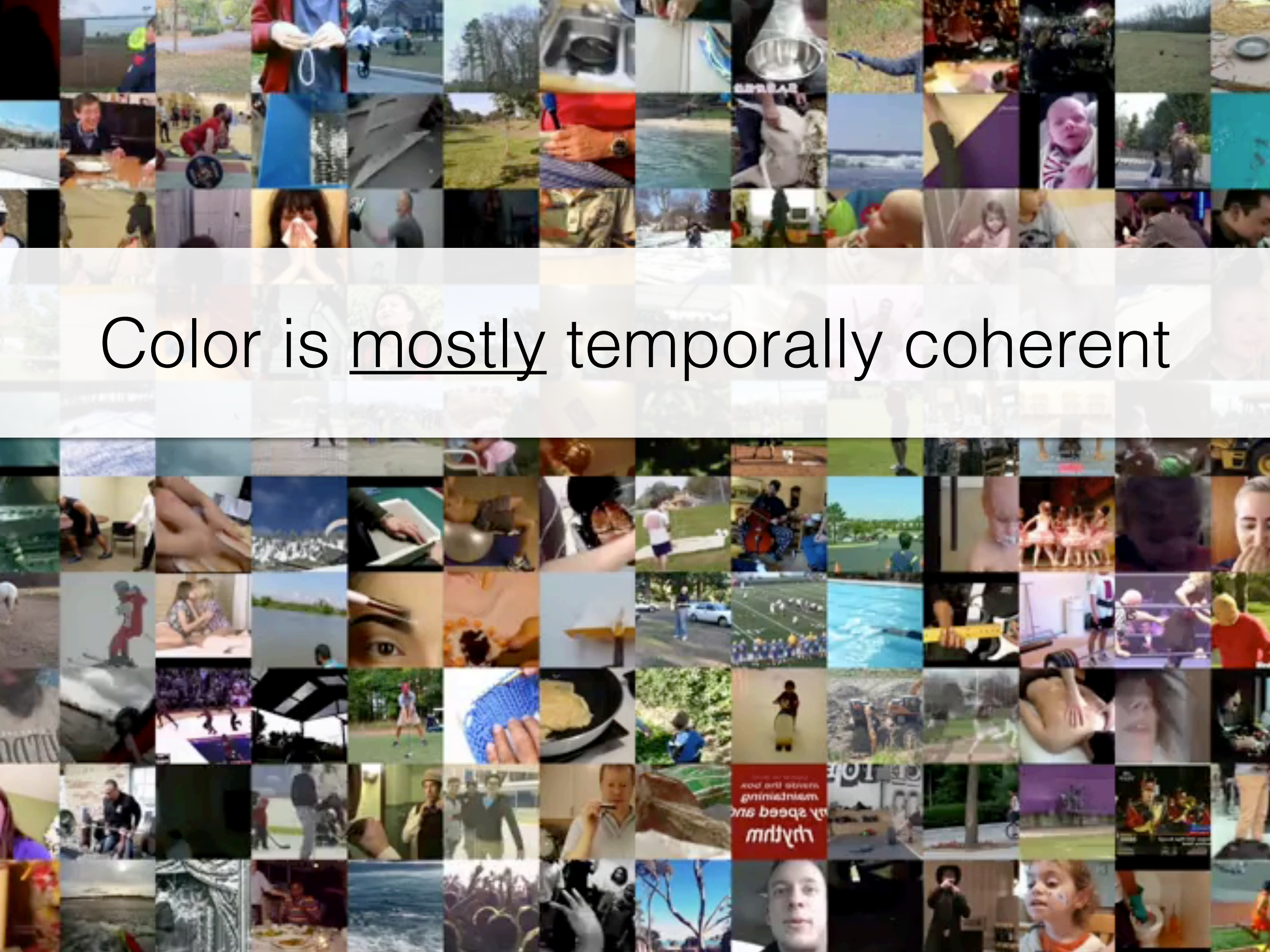
Edward Adelson, 1995



Obvious exceptions...





A large video frame grid, similar to the one in the first block, but with a central text overlay. The text "Color is mostly temporally coherent" is displayed in a large, black, sans-serif font, centered horizontally and vertically over the grid. The grid itself is composed of many small video frames, showing a variety of scenes and objects, with a color palette that is mostly consistent across the frames, supporting the claim of temporal coherence.

Color is mostly temporally coherent



# Self-supervised Tracking



Reference Frame



Gray-scale Video

What color  
is this?

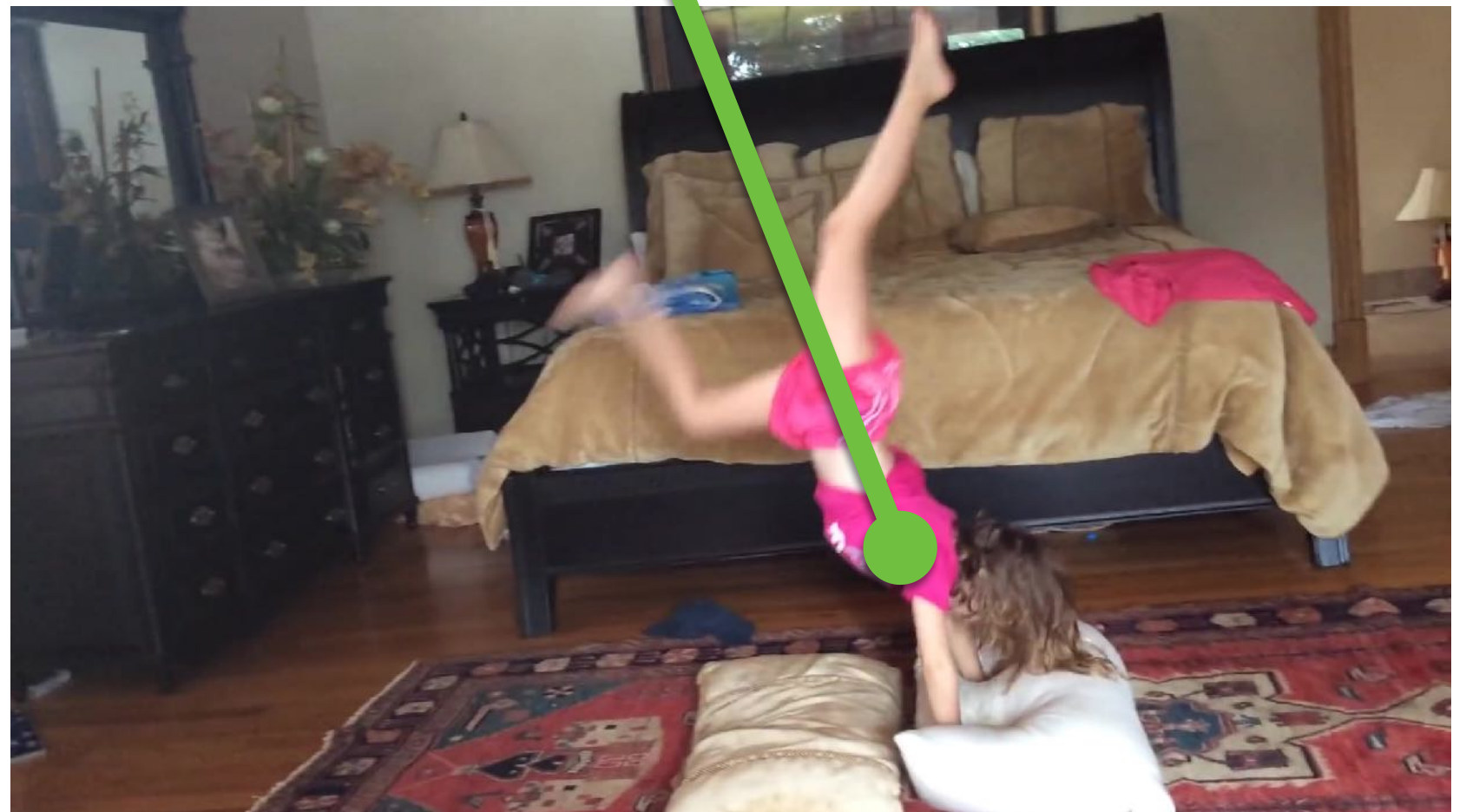




Where to  
copy color?



Want to be  
safe!





Where to  
copy color?





Color can be  
robust to  
occlusion

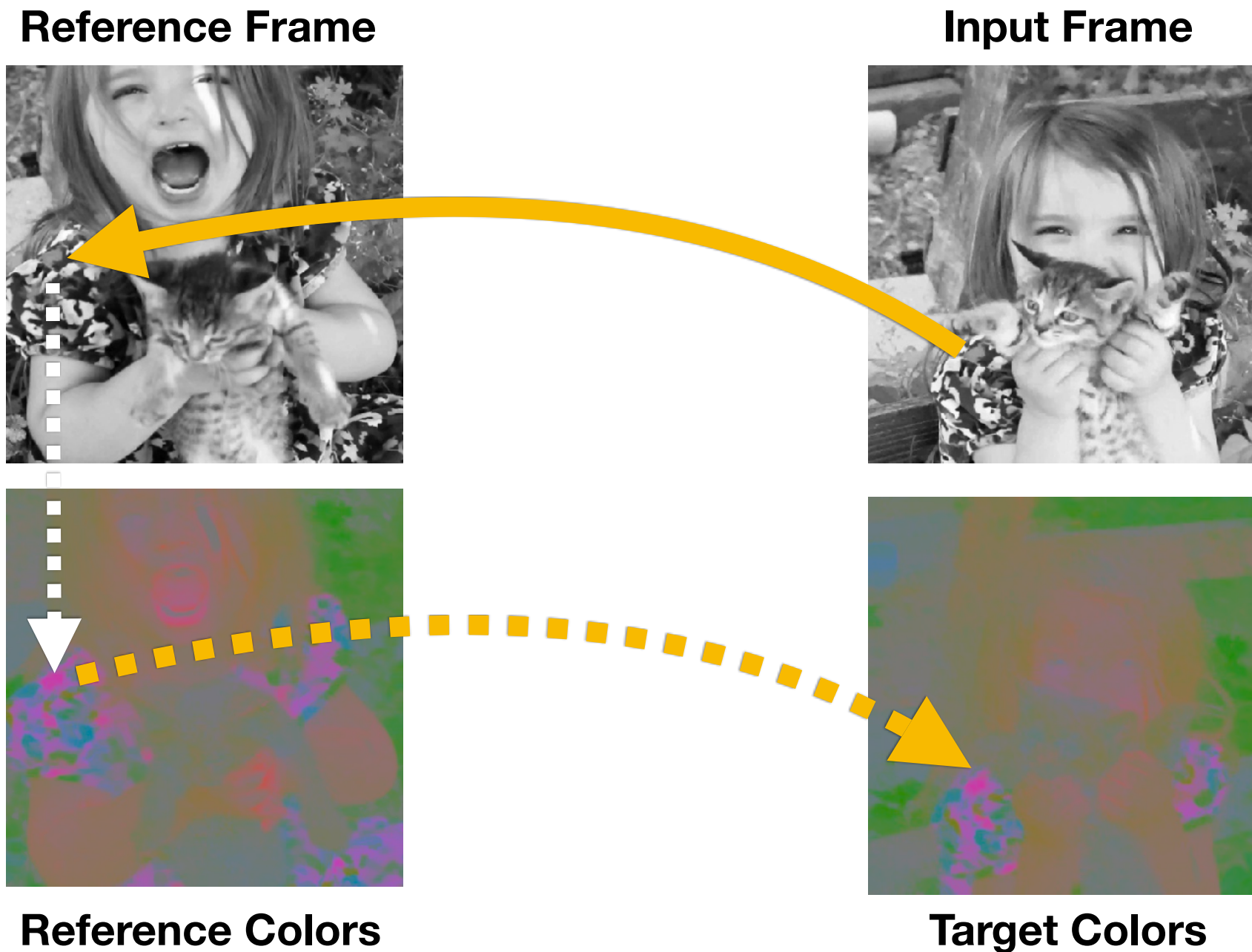


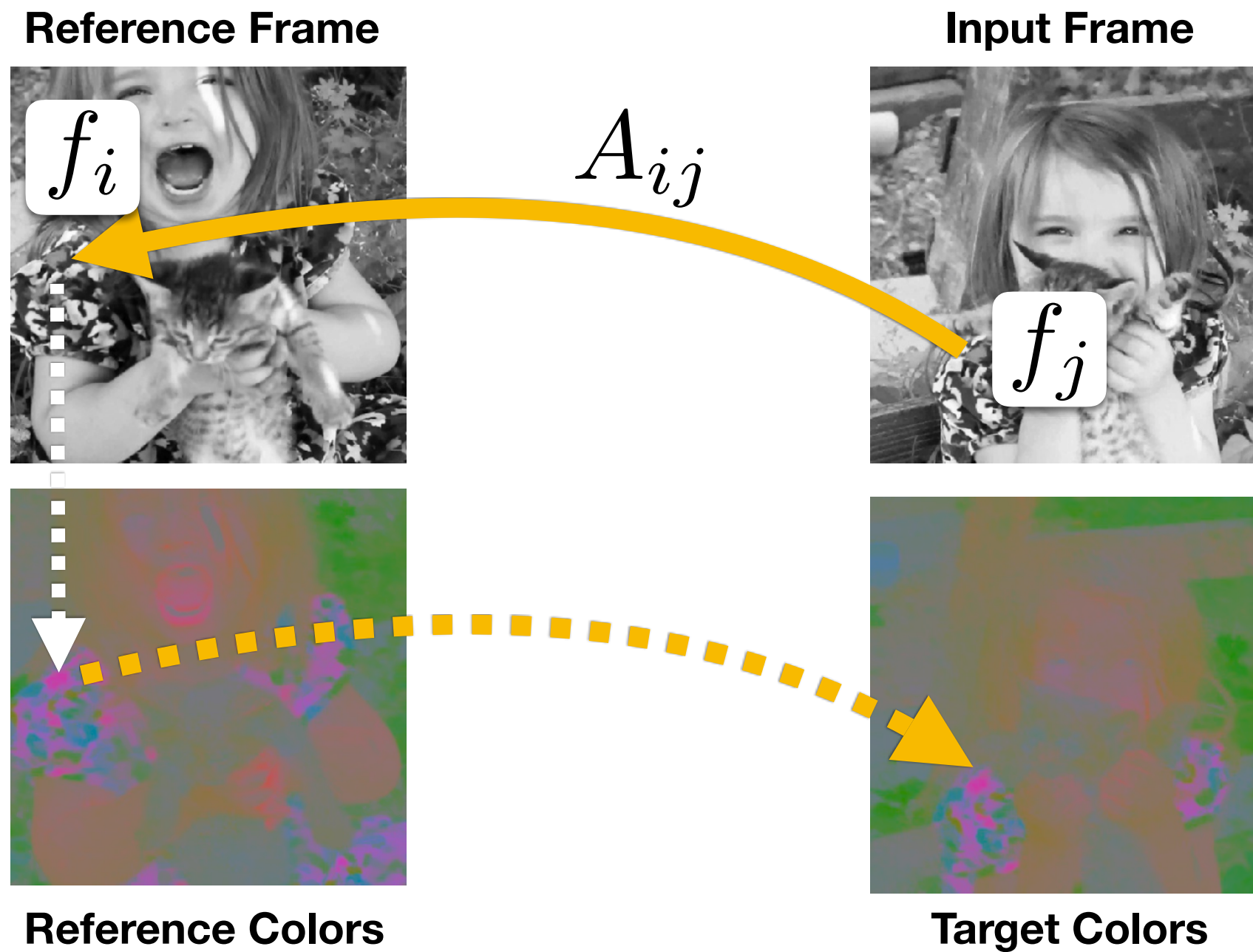
**Input Frame**



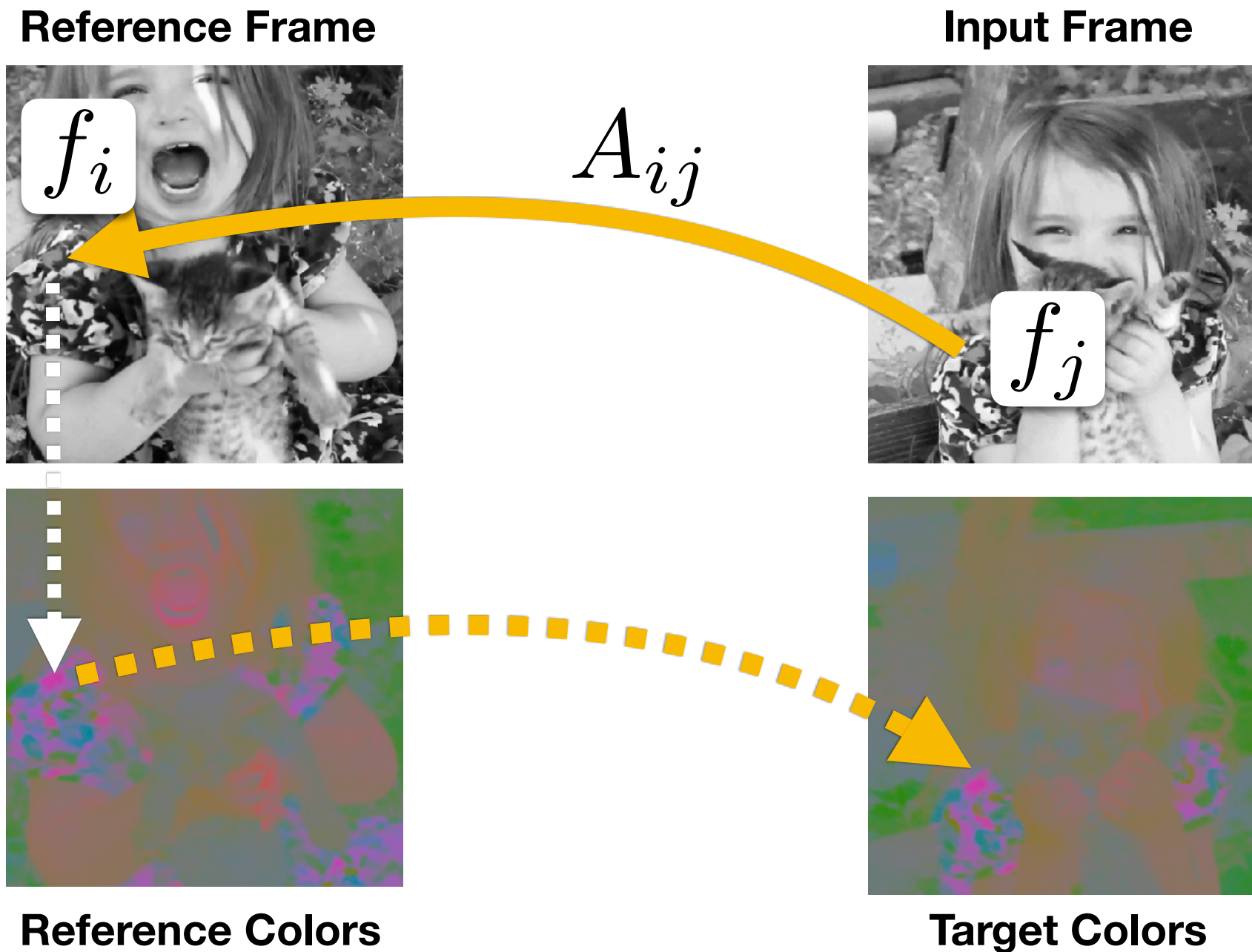


# Colorize by Pointing



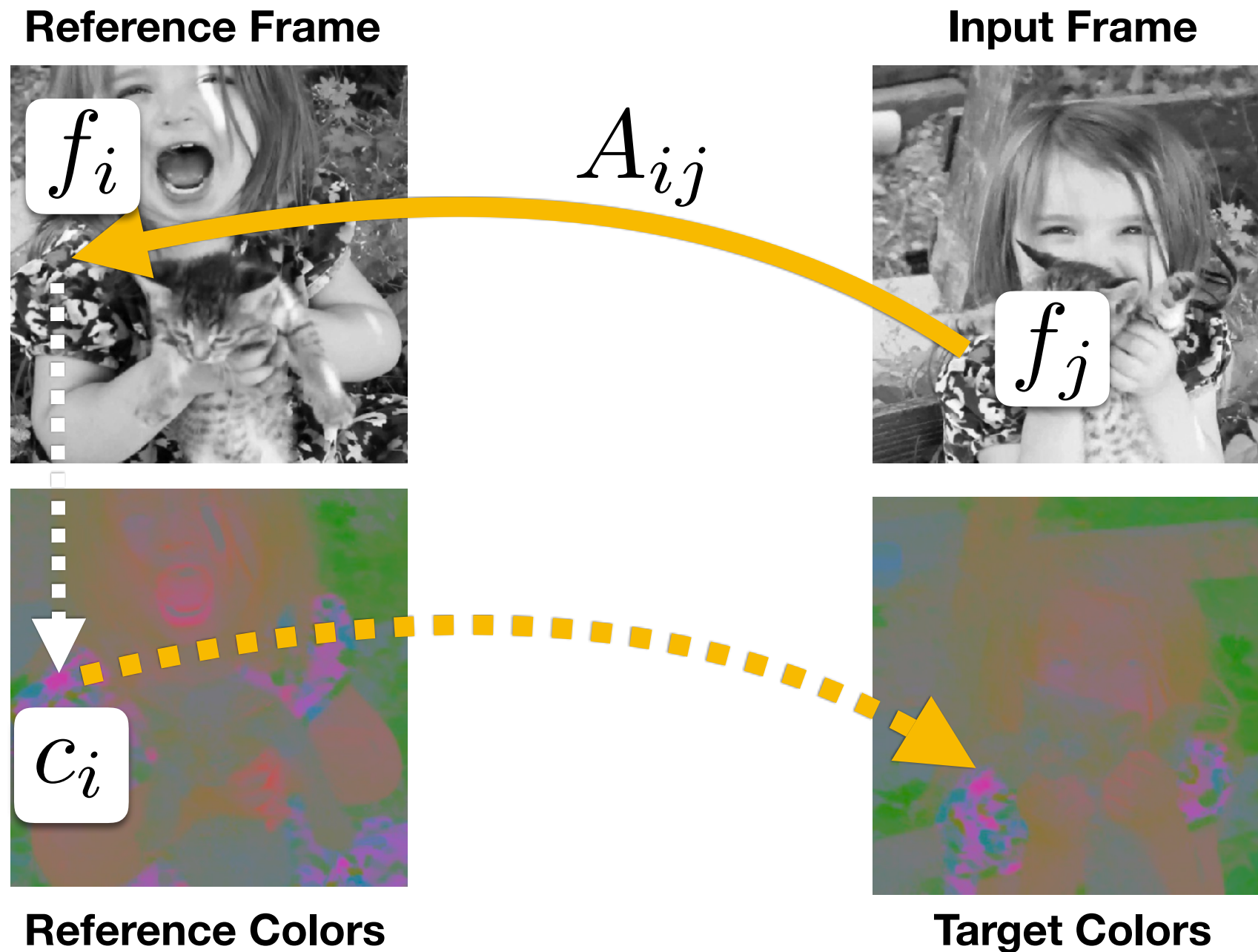


$$A_{ij} = \frac{\exp(f_i^T f_j)}{\sum_k \exp(f_k^T f_j)}$$

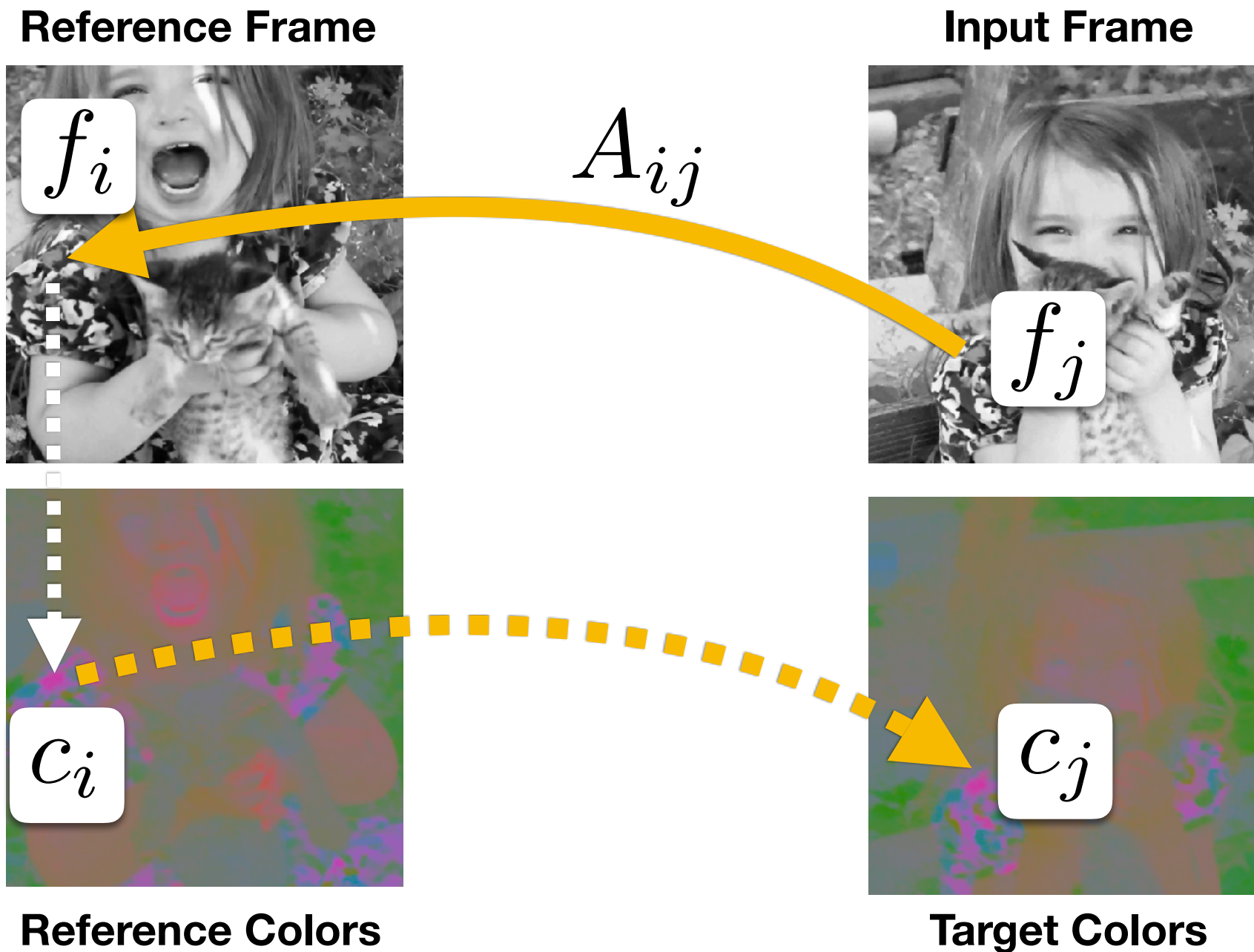




$$\hat{c}_j = \sum_i A_{ij} c_i \quad \text{where } A_{ij} = \frac{\exp(f_i^T f_j)}{\sum_k \exp(f_k^T f_j)}$$



$$\min_f \mathcal{L} \left( c_j, \sum_i A_{ij} c_i \right) \text{ where } A_{ij} = \frac{\exp(f_i^T f_j)}{\sum_k \exp(f_k^T f_j)}$$









**'ANNABELLE SERPENTINE DANCE' (1895)**





# Lumière Brothers

Inventors of  
motion picture, 1895

Inventors of first practical  
color camera, 1903

# Georges Méliès

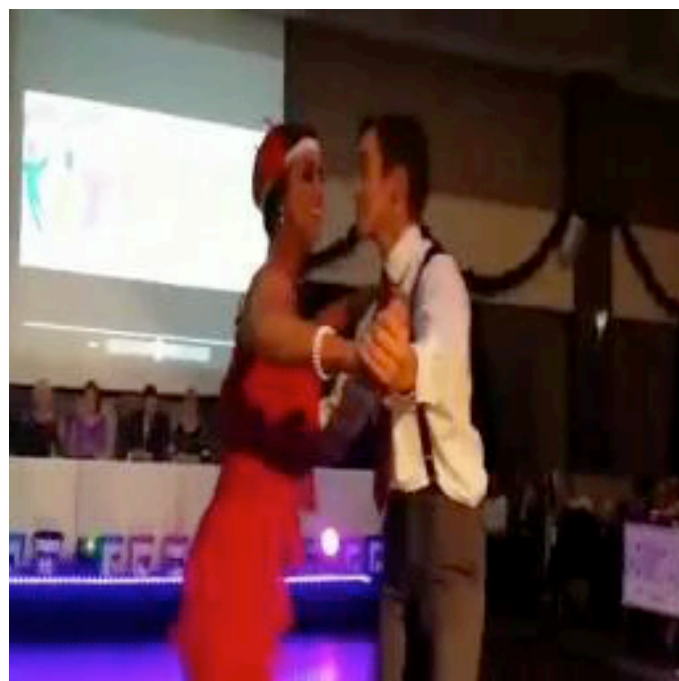
“Discovered” special  
effects, 1898





# Video Colorization

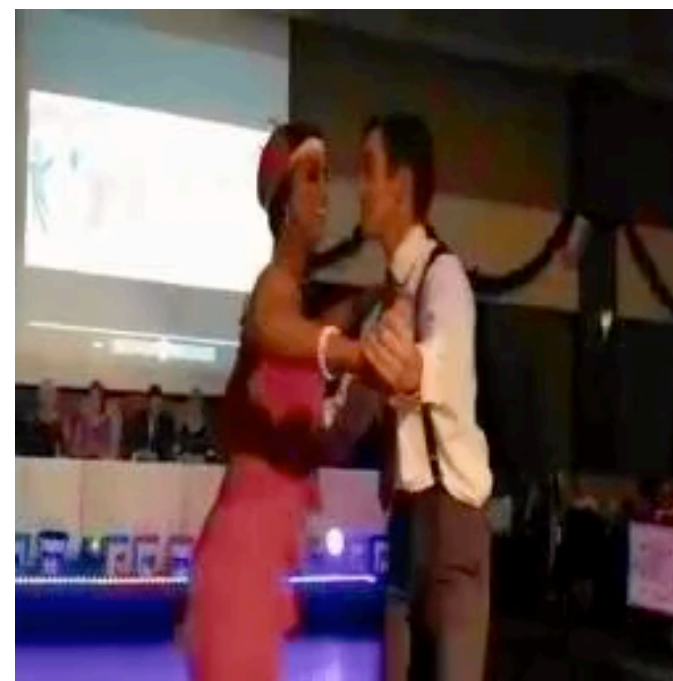
Reference Frame



Gray-scale Video



Predicted Color



# Video Colorization

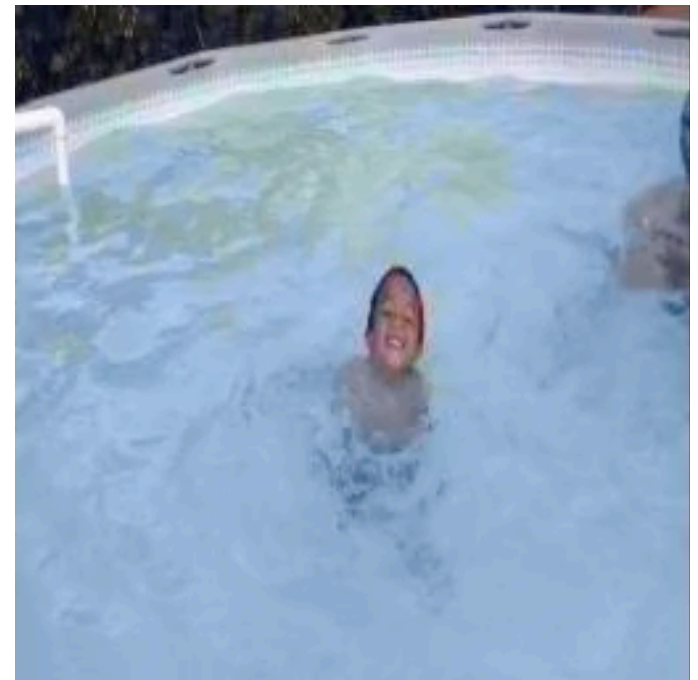
Reference Frame



Gray-scale Video



Predicted Color





# Tracking Emerges!

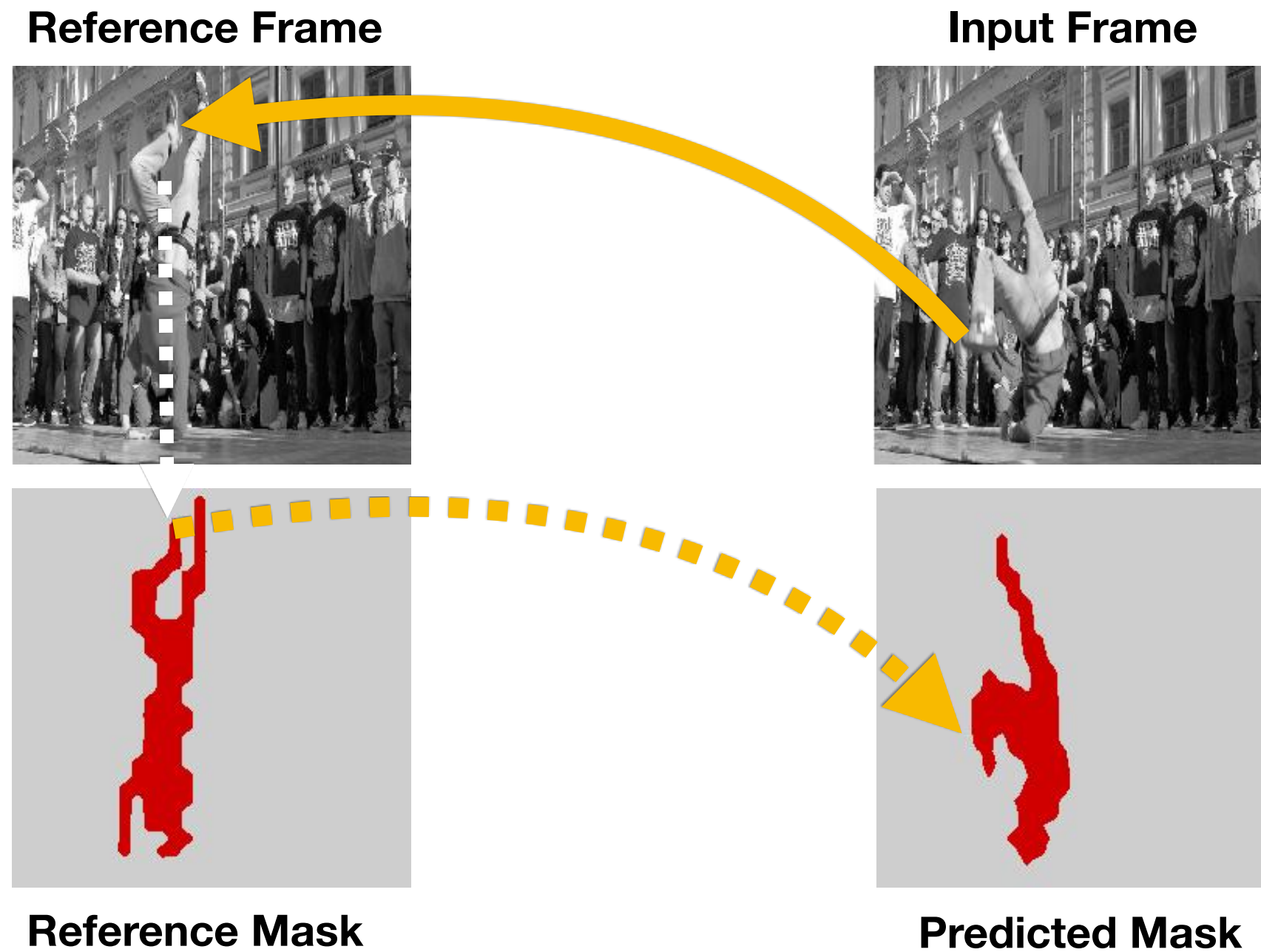
**Reference Frame**



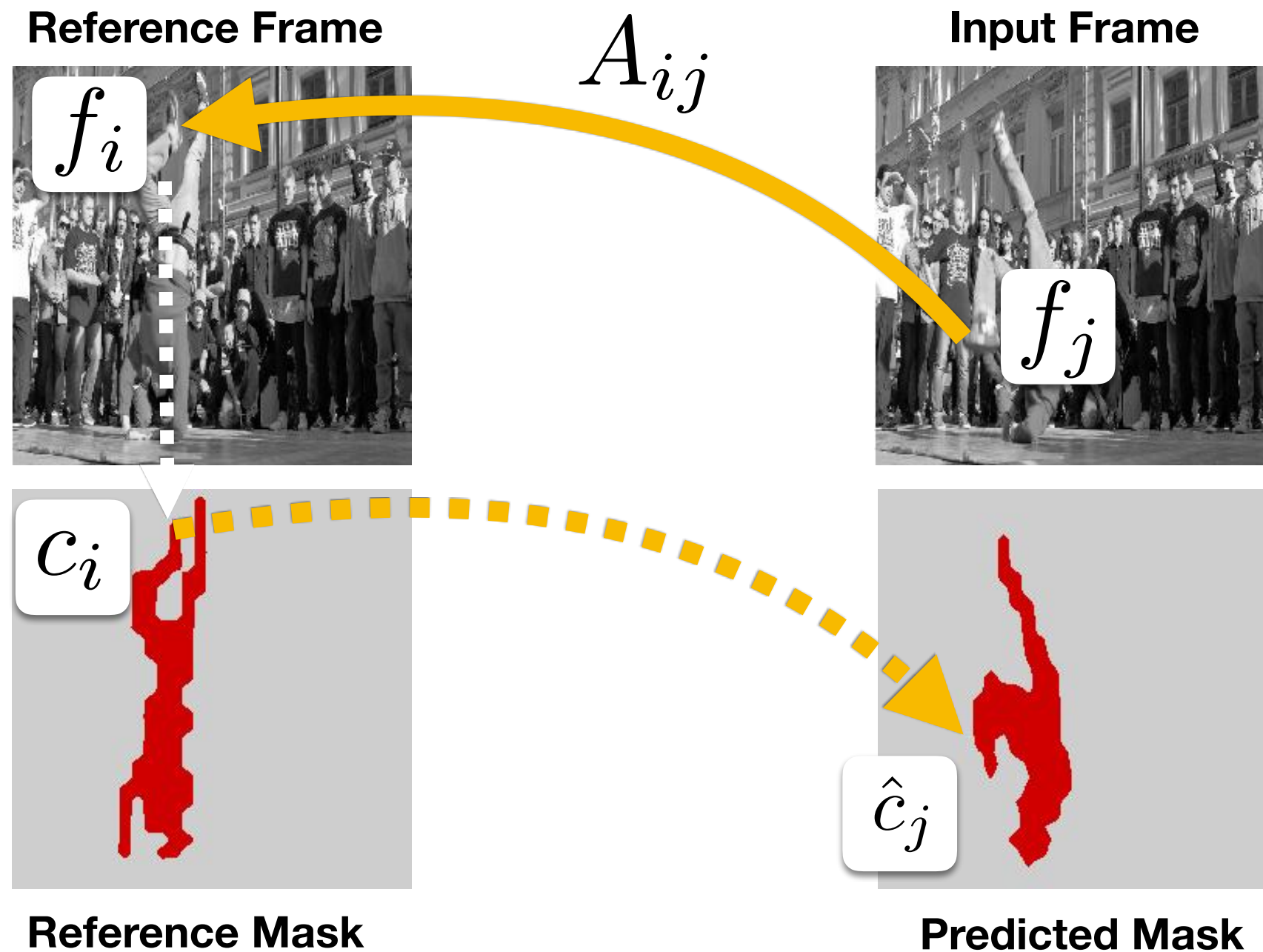
**Input Frame**



# Tracking Emerges!



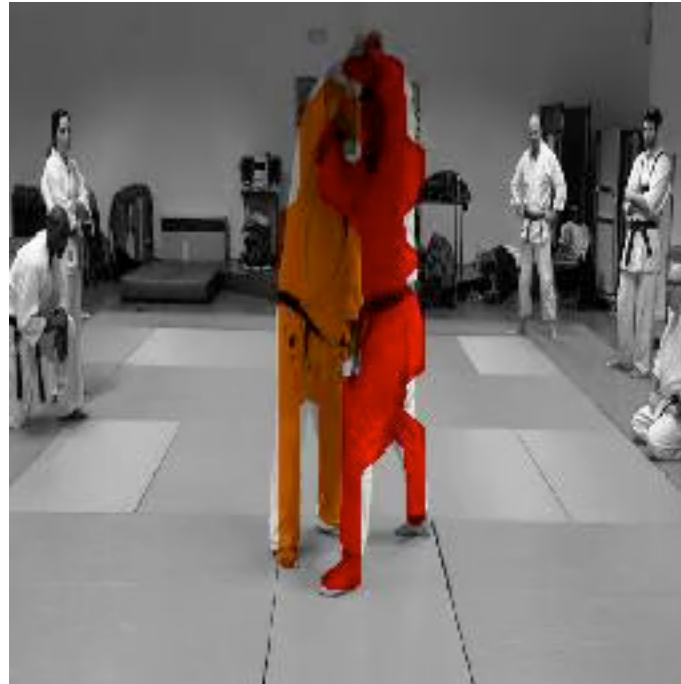
$$\hat{c}_j = \sum_i A_{ij} c_i \quad \text{where } A_{ij} = \frac{\exp(f_i^T f_j)}{\sum_k \exp(f_k^T f_j)}$$





# Segment Tracking Results

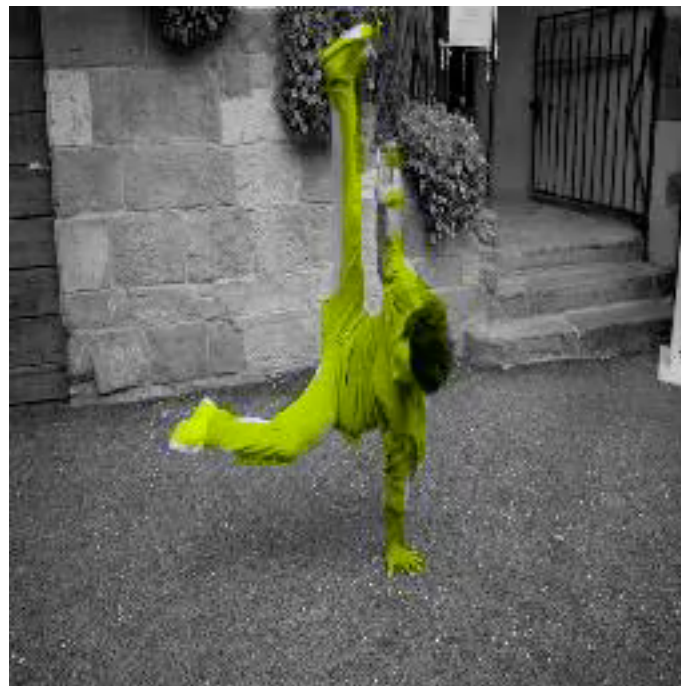
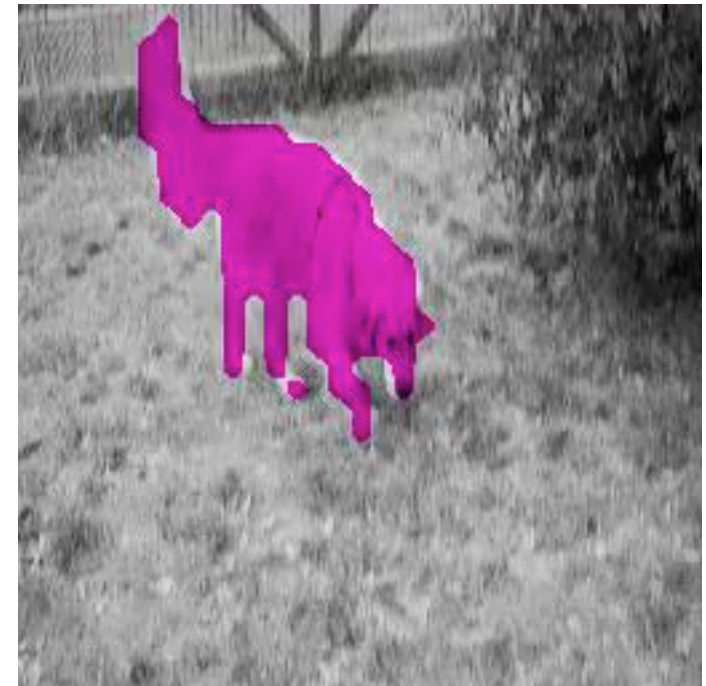
Only the first frame is given. Colors indicate different instances.





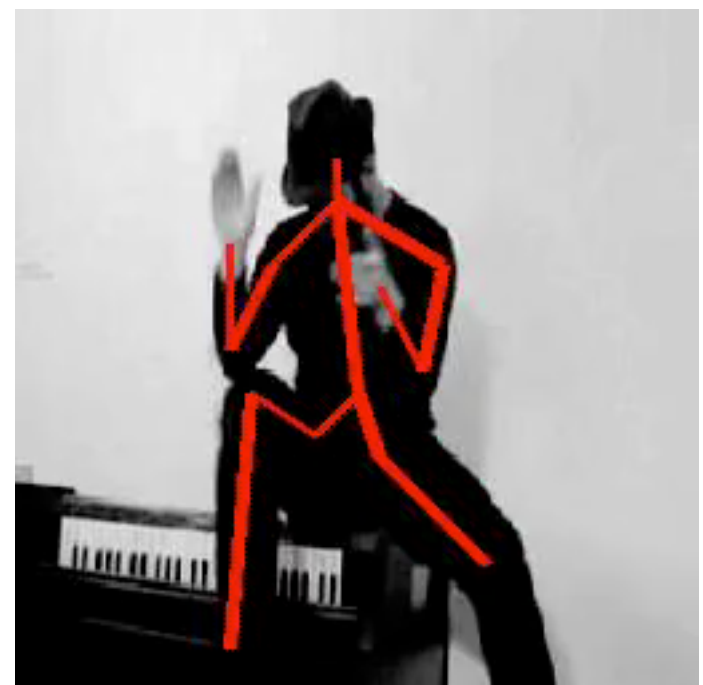
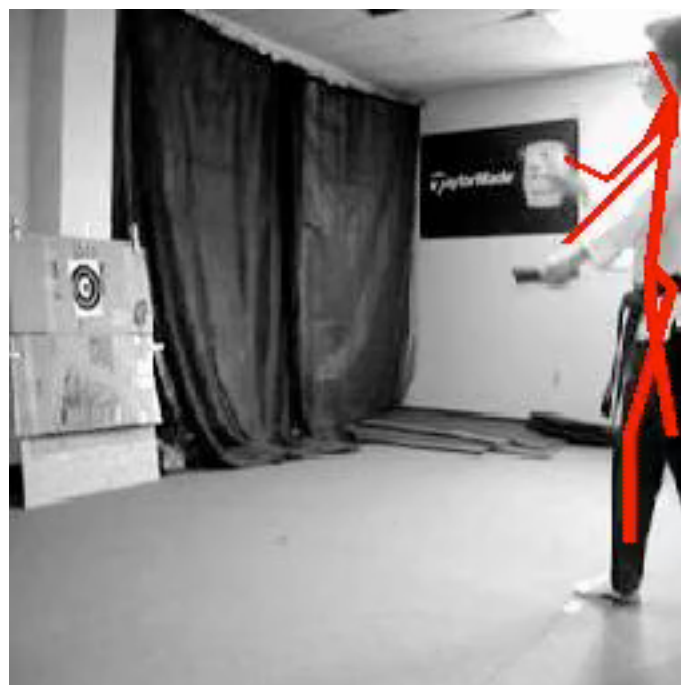
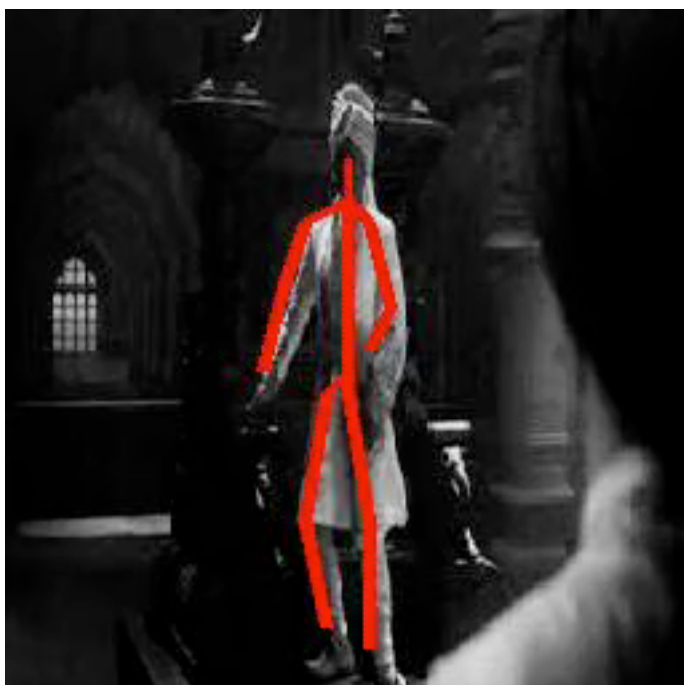
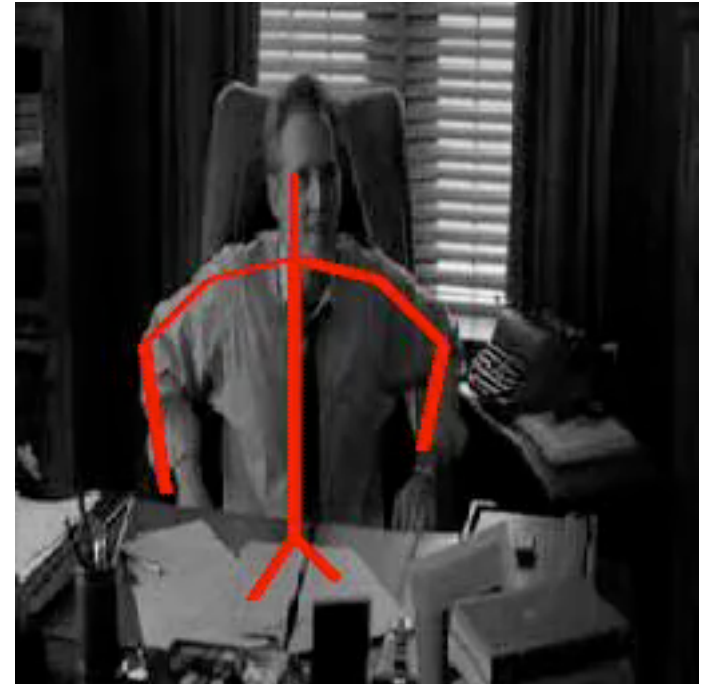
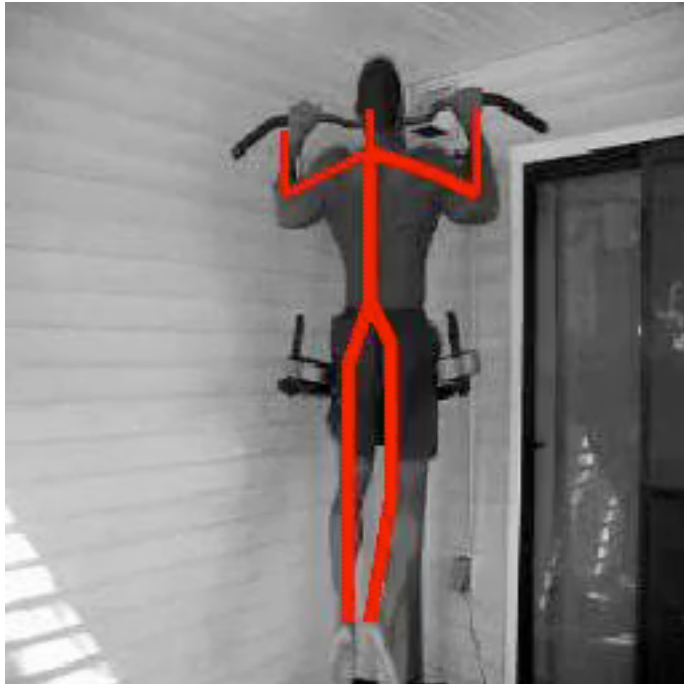
# Segment Tracking Results

Only the first frame is given. Colors indicate different instances.



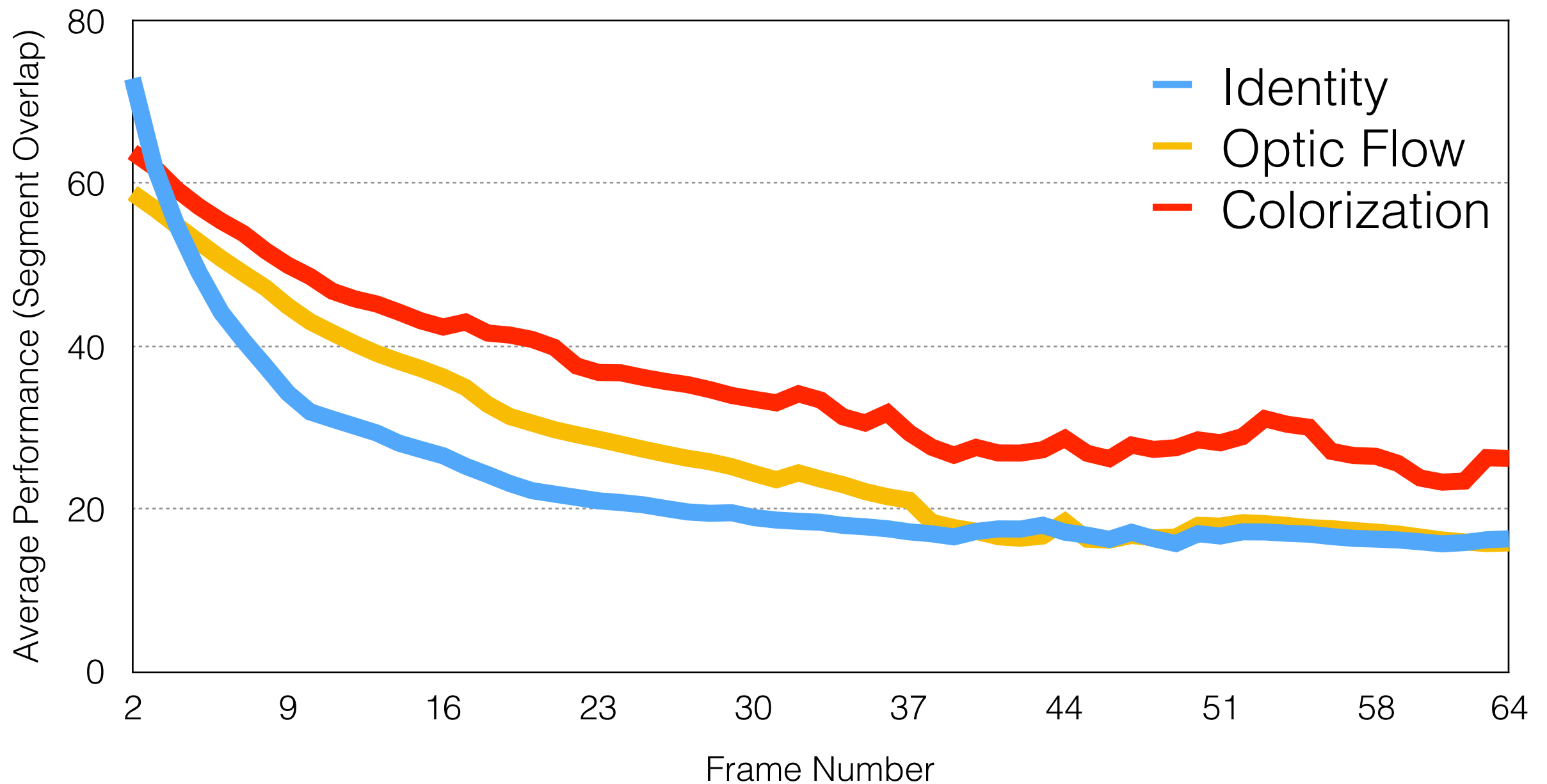
# Pose Tracking Results

Only the skeleton in the first frame is given.

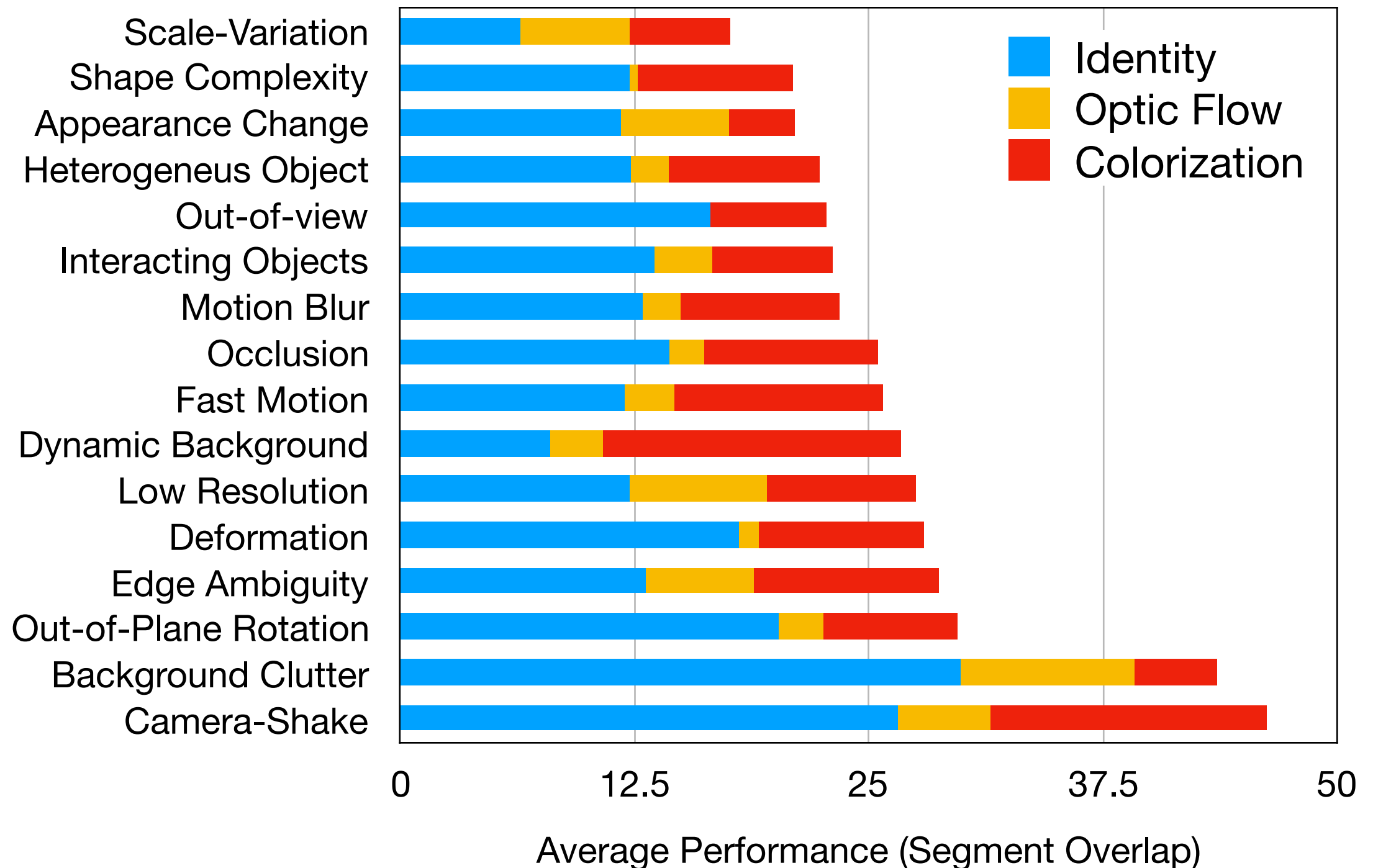




# Tracking Performance



# Tracking Performance



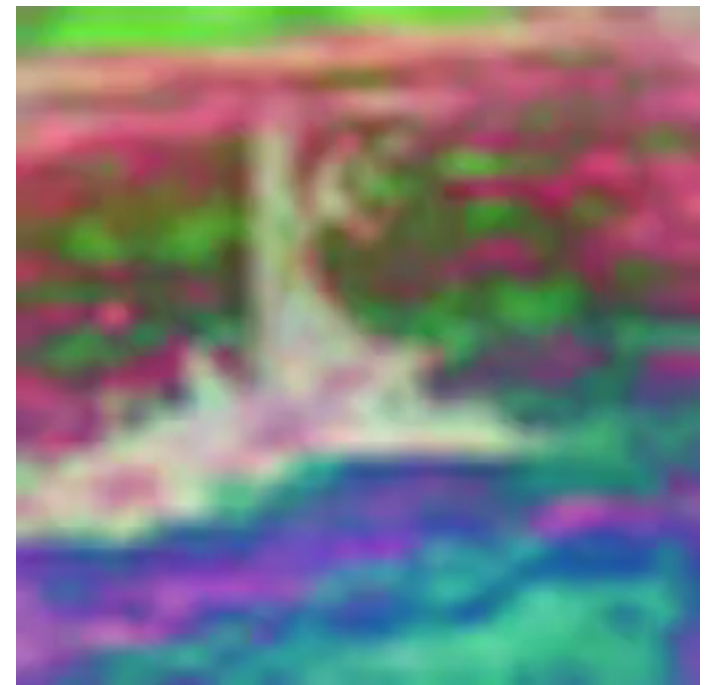
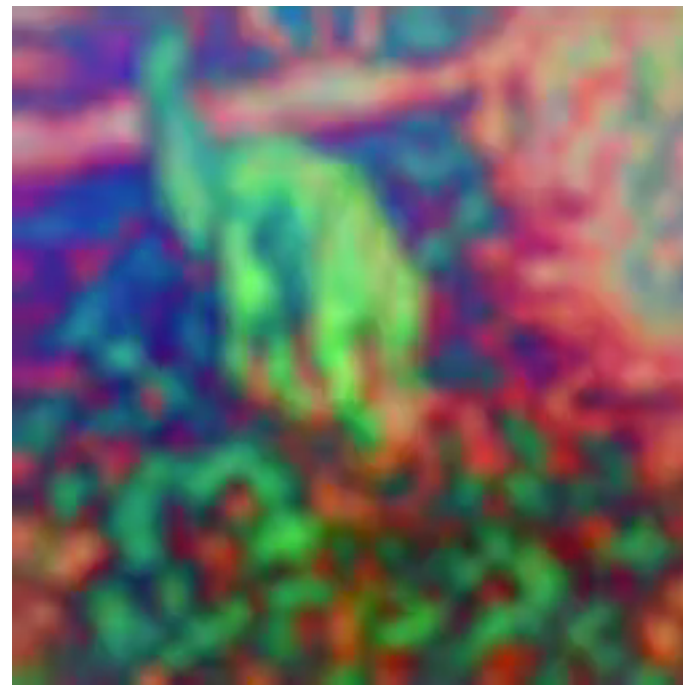
# Visualizing Embeddings

Project embedding to 3 dimensions and visualize as RGB

Original  
Video



Embedding  
Visualization





# Colorization and tracking fail together

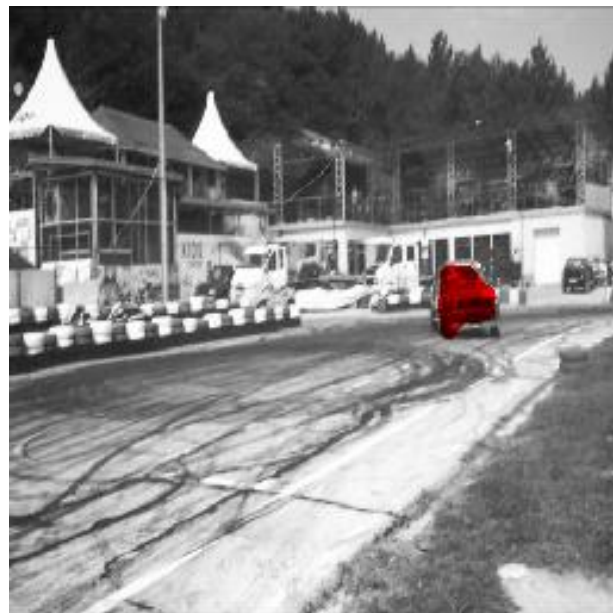
Reference  
Colors



Predicted  
Colors



Reference  
Mask



Predicted  
Mask

